Why Do IPO Underwriters Allocate Extra Shares when They Expect to Buy Them Back?

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Abstract

I argue that overallocation is used as a marketing strategy to increase the offer price and aftermarket price of an initial public offering (IPO). I show that, when there is weak demand, it can be optimal for the underwriter to oversell an issue and take a naked short position. The issuing firm benefits from a higher expected offer price. This is in spite of the fact that, in equilibrium, allocating more shares when there is weak demand requires greater underpricing when there is strong demand.

I. Introduction

Most of the literature on the allocation of IPO shares, as emphasized by Ritter and Welch (2002), has focused on who receives the allocated shares (see e.g., Brennan and Franks (1997), Booth and Chua (1996), Stoughton and Zechner (1998), Field and Karpoff (2002), Cornelli and Goldreich (2001), (2003), Rock (1986), Benveniste and Spindt (1989), and Sherman (2000)). Aggarwal (2000), Fishe (2002), and this study focus on a new dimension—how many shares are allocated. An overallotment, or green shoe, option allows underwriters to sell a maximum of 15% extra shares at the offer price. However, underwriters frequently allocate more than 115% of an IPO. Aggarwal (2000) reports that underwriters allocate on average 122.66% of the stated offer size for IPOs with an offer price less than the minimum of the original filing range (weak IPOs). Part of this short position can be covered by exercising the overallotment option, but a short position above 15% means that underwriters are committed to buy the incremental shares back in the aftermarket. In Aggarwal’s sample, the average naked short positions for IPOs with an offer price within the filing range and IPOs with an offer price above the filing range are, respectively, 1.05% and 0.25%. Thus,
underwriters are more likely to take naked short positions for IPOs where there is weak demand.

Why would underwriters allocate extra shares to investors when they expect to buy them back? In this paper, I argue that underwriters can increase an IPO’s aftermarket price by overallocating the issue in the premarket. I call this the path dependency hypothesis because the IPO’s aftermarket price depends on the premarket allocation. I incorporate this path dependency hypothesis into an information acquisition framework first proposed by Benveniste and Spindt (1989), and show that when demand is weak, it can be optimal for the underwriter to oversell an issue and take a short position beyond what can be covered by the overallotment option.

The path dependency argument—overallocation increases the aftermarket demand and the stock price—is a crucial assumption of the model in this paper. I offer four justifications for this assumption. First, investors who already own a security are more likely to hold it next period than those who must still purchase it. Thus, overallocation results in greater aftermarket demand due to this endowment effect. This phenomenon has been confirmed in numerous experiments (Kahneman, Knetsch, and Thaler (1990)). Second, the use of penalty bids, in which the lead underwriter takes back a syndicate member’s selling concession if its investors flip shares, makes it more likely that investors who are allocated shares in the premarket will hold the stock. Third, overallocation allows the underwriter to increase institutional investors’ average allocation, which makes it easier for those investors to reach their minimum position requirements in the IPO. Everything else equal, a large initial allocation at the offer price makes it more likely that an institutional investor will decide to accumulate, instead of sell, the stock. Finally, overallocation increases the stock’s investor base, resulting in better risk sharing and liquidity (Merton (1987) and Booth and Chua (1996)). This will lower the expected return and increase the stock’s value.

In the model, the underwriter distributes a new issue to a group of investors who agree to purchase up to a certain number of non-overpriced shares. The objective for the underwriter is to maximize its gross spread (the proportional commission) minus the potential costs to cover a short position if there is overallocation. The underwriter needs to collect information from the investors, and also needs to determine the extent of overallocation. Each investor receives a private signal that could be either good or bad, and the market price of the IPO is positively related to the total number of good signals. Investors who receive a good signal may want to report a bad signal so that they can get mispriced shares at a lower price. The underwriter has to provide underpriced IPO shares as the incentive for investors to truthfully reveal information. Such underpricing reduces the total proceeds and the underwriter’s commissions. To minimize underpric-

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1With the exception of index funds, a typical institutional investor holds a limited number of stocks in its actively managed portfolio. In order to amortize the fixed costs of following a stock, it is common for a money manager to have a minimum position size in mind. As a result, if an institutional investor is going to hold shares of a particular stock, it might insist on holding at least 40,000 shares.

2It is always optimal for the underwriter to presell the IPO up to at least 115% of the issue size because of the existence of the overallotment option. I take the 115% allocation as the base case. My discussion about overallocation focuses on the extra shares allocated beyond the overallotment option, i.e., the naked short position.
ing, it is optimal to underprice only hot IPOs for which demand is strong (i.e., there are many good signals) and allocate underpriced shares only to investors with good signals. Because hot IPOs are underpriced, the cost of buying back oversold shares at a higher price is greater than the increased commissions due to overselling. This cost removes any incentive that the underwriter has to oversell hot IPOs.

Because cold IPOs are neither underpriced nor overpriced, the underwriter does not incur costs to buy back oversold shares of cold IPOs. However, there is still a trade-off associated with overselling. On the one hand, the underwriter gains because it can increase the offer price since the market price is increased by overselling. On the other hand, the underwriter loses because overselling can also result in greater allocations for investors reporting bad signals. This provides greater incentives for investors with good signals to lie because the benefit that an investor obtains by falsely reporting a bad signal is the product of the allocation for an investor reporting a bad signal and the mispricing caused by the false report. The underwriter has to further underprice hot IPOs to provide sufficient incentives for truthful information revelation. If the increase in underwriting revenues due to overselling a cold issue is greater than the required increase in underpricing on a corresponding hot issue, it is then optimal for the underwriter to oversell the cold issue.

The results of the model are consistent with Aggarwal’s (2000) finding that only weak IPOs are oversold. The results are also consistent with Aggarwal (2000) and Ellis, Michaely, and O’Hara’s (2000) finding that underwriters break even on short covering of oversold shares.

Alternative explanations for overselling weak IPOs exist in the literature. Wilhelm (1999) suggests that, by preselling extra shares, when underwriters buy back these shares, they can create buying power to support IPOs in the aftermarket. He does not explain, however, why some of the parties who are allocated shares would not otherwise have been buying in the aftermarket. Fishe (2002) argues that the underwriter oversells an IPO because it can buy back oversold shares at a lower price. He argues that the offer price can be inflated by artificial demand created by stock flippers. When these stock flippers flip their shares in the aftermarket, the price will fall, creating profits on the short position.

The paper is organized as follows. I describe the basic setup of the model in the next section, and discuss the path dependency hypothesis in detail in Section III. The major results of the model are given in Section IV. Empirical evidence supporting the model, as well as other alternative explanations, is discussed in Section V. Section VI concludes the paper.

II. The Set Up

The underwriting process of an IPO using bookbuilding starts with the issuing firm signing a “letter of intent” with the lead underwriter, and the lead underwriter’s activities, such as market making and price support, go on into the aftermarket trading of the stock. I model the underwriter’s overselling behavior.
during an IPO’s premarket information collection, bookbuilding, and share allocation process. I adopt the information acquisition framework first proposed by Benveniste and Spindt (1989). In addition to the issuing firm and the underwriter, the IPO process involves $I$ institutional investors (see Appendix I for a notation list). I do not include individual investors in the model. Although individual (small) investors are usually allocated shares, their allocation is more or less a fixed proportion of the total shares offered across different IPOs (Hanley and Wilhelm (1995) and Aggarwal, Prabhala, and Puri (2002)). Incorporating individual investors in the model would allow the underwriter to reduce underpricing, but the main implications of the model would not change.

Let the total number of shares for the IPO be fixed at $Q$, which includes the green shoe option. The green shoe option gives the underwriter a no-lose situation. If the aftermarket price rises, the underwriter can exercise the option to cover the 15% extra shares, and earn the underwriting discount on the extra shares sold. If it is necessary for the underwriter to support the issue in the aftermarket, the underwriter can then cover the 15% extra shares using shares bought in the aftermarket at a price no higher than the offer price. So, the size of the offering $Q$ is the total number of shares offered plus the overallotment option. The number of shares that each investor can take lies in $[\bar{I}, \bar{I}]$, or it will be zero. I assume that $I \times \bar{I} < Q$, and $I \times \bar{I} > Q$.

Each investor independently receives a signal, which can be either good or bad. The probability of a good signal is $p$, and the probability of a bad signal is $1 - p$. The signal is private before it is reported, and once reported to the underwriter, it becomes common knowledge. As a result, the underwriter cannot overstate the number of good signals as modeled in Benveniste, Busaba, and Wilhelm (1996). Furthermore, I assume that investors report their private signal simultaneously to rule out information cascades (Welch (1992)). State $i$ refers to the situation of having $i$ good signals. The probability for state $i$ is

$$
\pi_i = \binom{I}{i} p^i (1 - p)^{I - i},
$$

since each signal is an independent draw from a binomial distribution.

As is typical of firm commitment IPO using bookbuilding, the underwriter’s task is to presell all $Q$ shares, or more than $Q$ shares if the underwriter takes a short position. Denote the total number of shares the underwriter presells in state $i$ as $\theta_i Q$, where $\theta_i \geq 1$ represents the preselling scalar. If $\theta_i > 1$, the underwriter takes a naked short position of $(\theta_i - 1)Q$ shares, and needs to buy back those shares in the aftermarket at the then current market price.

The underwriter determines the offer price and the number of shares allocated to each investor. The number of shares allocated to each investor with a good (bad) signal is $q_{g,i}$ ($q_{b,i}$). Note that $q_{g,i}, q_{b,i} \in [\bar{I}, \bar{I}]$. Furthermore, I assume that the underwriter only allocates $q_{b,i}$ shares to investors with bad signals with

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4See Maksimovic and Pichler (2002) for a thorough treatment of allocation between institutional and retail investors. They also allow retail investors to be possibly informed and combine Rock’s (1986) winner’s curse problem with Benveniste and Spindt’s (1989) information gathering from institutional investors.
probability \( \lambda_i \in [0, 1] \). That is, the expected allocation for an investor with a bad signal is \( q_{b_i, \lambda_i} \).

I assume that all parties are risk-neutral. The underwriter earns a fixed proportional fee, \( c \), of the gross proceeds. The investors only accept the shares allocated to them if the offer price is no higher than the expected market price (the individual rationality constraint). Let \( OP_i \) and \( P_i \) represent the offer price and the market price, respectively, in state \( i \). The expected gain for the underwriter is then as follows,

\[
(1) \quad \sum_{i=0}^{I} \pi_i [c OP_i Q - (\theta_i - 1)(P_i - OP_i) Q].
\]

Equation (1) is the objective function for the underwriter. The first term in brackets is the underwriting discount from the issue in state \( i \), and the second term is the loss from buying back oversold shares in that state.

The timing of the game is as follows. When the game begins, the investor pool is determined exogenously. The underwriter provides the preliminary prospectus to investors. At this point, the characteristics of investors, the probability of a good or bad signal, and the number of shares \( Q \) become common knowledge. The investors then receive their private signals. The underwriter declares the allocation schedule (including \( q_{g_i}, q_{b_i, \lambda_i}, q_{b_i}, \) and \( \lambda_i \)) and the offer price for each state \( i \). The informed investors report their signals to the underwriter, and the underwriter then determines the offer price and the number of shares allocated to each investor as declared. The game is then over.

III. The Effects of the Preselling Scalar \( \theta_i \) and the Path Dependency Hypothesis

Following Benveniste and Spindt (1989), I assume that the IPO’s aftermarket price depends on the number of good signals \( i \). In addition, I assume that the aftermarket price is also related to the preselling scalar \( \theta_i \). That is,

\[
(2) \quad P_i = P - (1 - i) \alpha + f(\theta_i).
\]

Note that \( f(\theta_i) \) captures the effect that overallocation has on the aftermarket price. Since \( \theta_i \) is a premarket variable, equation (2) implies that the IPO price is path dependent. In this section, I examine why and how the premarket allocation can affect the aftermarket price.

During the road show period, the underwriter and the issuing firm usually fly all over the country to give presentations to groups of institutional investors. The underwriter also holds one-on-one meetings with important money managers such as Fidelity. One purpose of this intensive road show is to gather information from investors to gauge market demand, and this is the intuition that the information

\[\text{By introducing the probability } \lambda_i, \text{ I effectively relax the quantitative restriction on the number of shares investors with bad signals can take from } [0, I] \text{ to } [0, I]. \text{ This is for the convenience of expressing the results. It also captures the fact that the underwriter may treat each investor who reports a bad signal differently.}\]
acquisition part of the model tries to capture. However, the road show is also an intensive marketing campaign, and one important goal of this campaign is to stimulate demand and shift up the demand schedule for the stock (Ritter (2003)). I argue that overallocation (choosing \( \theta_i > 1 \)) is part of the underwriter’s marketing strategy.

Everything else equal, the underwriter can use overallocation to increase the aftermarket price in two ways. First, given a downward sloping demand curve, the underwriter can increase the aftermarket price if it can increase the aftermarket demand for the stock.\(^6\) Second, if increasing the investor base for the stock results in better risk sharing and liquidity (Merton (1987)), then overallocation that expands the investor base can lower the expected return and hence increase the stock price.

A. Overallocation Increases the Aftermarket Demand and Shifts the Demand Curve Upward

The endowment effect refers to the phenomenon that people are more likely to hold an item if it is already in their possession than if they have to buy it. For example, in an experiment reported by Kahneman, Knetsch, and Thaler (1990), the median price for students to give up a Cornell coffee mug in their possession is $5.25, although they would only pay $2.25 to buy one. This behavior is consistent with prospect theory (Kahneman and Tversky (1979)), which asserts that people calculate gains or losses with respect to a reference point, and they value the gains or losses using an asymmetric value function. The reference point is path dependent, and usually is the status quo (this is why the endowment effect is also called the status quo bias). The value function is concave in the gain domain and convex in the loss domain. Prospect theory postulates that people are also more sensitive to loss, which is reflected in the more negative slope of the convex part of the value function near zero. In the coffee mug example, a seller who already has possession views the coffee mug as a loss, while the buyer views it as a gain. The value function for the loss part is much steeper for small losses, and this is why the asking price for the coffee mug is much higher.

Evidence suggests that the endowment effect also exists in securities markets (see e.g., Shefrin and Statman (1985), Odean (1998), and Loughran and Ritter (2002)). Given that investors have participated in the premarket bidding during the underwriter’s marketing campaign, overselling the stock would allow more investors to own the stock and hence change the status quo for those investors who otherwise would not get shares. Because of the endowment effect, the change of the status quo makes an investor more likely to hold the stock than if he has to buy it in the aftermarket. By choosing \( \theta_i > 1 \), the oversold shares, \( (\theta_i - 1)Q \), will help to increase the number of investors who hold the stock in the aftermarket.

\(^6\)Cornelli and Goldreich (2001), (2003) provide evidence showing that investment banks’ order books suggest a downward sloping demand schedule for IPOs in their samples. Kandel, Sarig, and Wohl (1999) present evidence of negatively sloped demand curves in IPO auctions. Information-based theoretical models such as Benveniste and Spindt (1989) and Rock (1986) also imply a downward sloping demand curve for IPOs because all these theoretical models assume differential information among investors. Some other IPO-related theoretical models, such as Fishe (2002), explicitly assume a negatively sloped demand curve.
aftermarket. Everything else equal, this will increase the aftermarket demand and shift up the demand curve.

Some characteristics of the institutional setup of the IPO selling process also make overselling desirable. The first feature is the use of penalty bids, in which the lead underwriter takes back the selling concession from a syndicate member if its investors flip shares. It is plausible that the money involved is of no real importance for some syndicate members and fund managers. However, the damage to the business relationships between the underwriters and the involved institutional investors can be disastrous—those investors who flip their shares may not be able to participate in attractive future issues. Thus, some investors may feel obligated to hold the stock if they are allocated shares.

Another institutional feature that makes overallocation important in increasing the aftermarket demand is the minimum position requirement of institutional investors’ holding of an IPO. A typical institutional investor has a minimum position size in mind in buying IPOs. As a result, if the institutional investor is going to hold any shares in a stock, it might insist on holding at least 40,000 shares. In making its decision as to whether a given stock is worth holding, the money manager compares its assessment of value with the average cost of the shares, commonly known by practitioners as the value weighted average price (VWAP). This makes it very important for the underwriter to try to make an institutional investor’s allocation reach some threshold. For example, if there is a target holding of 40,000 shares, an allocation of 5,000 shares is likely to result in selling, whereas an allocation of 25,000 shares is likely to result in the purchase of 15,000 more shares. Everything else equal, overallocation helps the underwriter to meet the minimum position requirements. This in turn helps to increase the aftermarket demand.

B. Overallocation Decreases the Expected Return for the Stock

Investors tend to invest in stocks with which they are familiar, and ownership of a stock is one of the best ways for an investor to know the stock. Merton (1987) models the effects of such investors’ recognition in a static setup. Under the assumption that investors only invest in stocks they know and not all investors know all stocks in the economy, Merton shows that, at such an incomplete information equilibrium, a stock’s price is lower than the price at the full information equilibrium. Merton shows that the higher the degree of investors’ recognition, the higher the stock price.

In Merton’s model, as long as an investor “knows” a stock, he will consider it whenever he chooses his optimal portfolio. On the surface, this would suggest that, no matter what the underwriter does with overselling, the number of investors who know the IPO is \( I \) as is assumed in the model. This would be true if institutional investors did not have a minimum position size and would always consider a stock in their portfolio optimization as long as they once considered the stock. The word “know” in Merton’s model means that an investor constantly considers the stock in his portfolio optimization. For IPOs, some of the investors

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7 See the *Wall Street Journal* article, “Some Initial Stock Offers Continue to Produce Fat, Quick Profits, but Money Managers are Wary,” (Dec. 15, 1992) for some examples.
who participate in the premarket may never consider the stock if they are not allocated shares. Furthermore, it is conceivable that an investor who is allocated shares and hence has owned the stock will behave differently from an investor who is not allocated shares and hence has never owned the stock. The former investor, even if it sells the stock soon in the aftermarket, is more likely to invest in the stock if opportunity comes. So overselling is helpful in increasing the number of investors that participate in the aftermarket risk sharing of the issue. As suggested by Merton, such risk sharing can lower the expected return and increase the market price.

C. The Effect of Overallocation and Its Relationship to the Supply and Aftermarket Flipping

To capture the effect of overselling on the IPO’s aftermarket performance, I assume \( f(\theta_i) = m(1 - (1/\theta_i)) \). Consequently,

\[
P_i = \bar{P} - (1 - \bar{\theta}) \alpha + m \left( 1 - \frac{1}{\theta_i} \right).
\]

Note that \( m \) measures the effect that preselling has on the market price, and that \( \partial P_i / \partial \theta_i = m/\theta_i^2 > 0 \), and \( \partial^2 P_i / \partial \theta_i^2 = -2m/\theta_i^3 < 0 \). That is, for \( \theta_i \geq 1 \), increasing overallocation (\( \theta_i \)) increases the market price, but at a decreasing rate.

One might argue that overallocation will also increase the number of investors who sell shares, which would depress the aftermarket stock price, contrary to the argument that I have put forth. Note, however, that there are two types of supply and demand: flow and stock. Flow supply and demand are measured over a time interval, and they are likely to fluctuate. Stock supply and demand are measured at a point of time and are relatively stable over time. In the model, \( P_i \) is the equilibrium price for a period during which the stock supply and demand remain unchanged. Since all oversold shares will be bought back by the underwriter, overallocation will not increase the stock supply and depress the equilibrium stock price. Second, it is true that overallocation increases the flow supply, which may cause downward pressure on the stock price. However, this effect is temporary, and can only cause some price fluctuation that should not change the equilibrium price. Furthermore, even this price pressure may be offset because of the increase of flow demand when the underwriter buys back shares to cover the short position.

A related issue is flipping, which refers to investors selling the allocated shares within a few days in the aftermarket for quick profits. Given the perception that IPOs have extremely high turnover rates during first-day trading, it may seem at odds with my argument that overallocation increases demand. However, as discussed above, flipping affects flow supply and demand—it will not have any fundamental effect on the stock price. Furthermore, the extremely high turnover for hot IPOs during the Internet bubble period (1999–2000) is not representative. Historically, for IPOs with a first-day return below 10% (non-hot IPOs), the average first-day turnover rate is below 50% even with the double counting problem (Table 5, Loughran and Ritter (2003)). If I divide that number by two, the number
of investors who flip their shares is less than one fourth. Even this is an overestimate because the turnover rate is calculated using a 100% allocation in the denominator, rather than 115% or higher. Also, the high turnover rate for the first few days does not imply high flipping. Aggarwal (2003) reports that, despite the high turnover rates, flipping only accounts for 15% of shares offered. A lot of shares simply change hands more than once.

IV. Optimization with the Preselling Factor

In this section, I first describe the underwriter’s optimization program. The solutions of the program and interesting features of the allocation process are discussed in Propositions 1 through 5. Proposition 6 states that both the underwriter and the issuing firm benefit from overallocation. All proofs are provided in Appendix II. Detailed numerical examples are provided in Appendix III to illustrate the intuition of the model.

A. The Program

The program for the underwriter, denoted as PATH, is as follows (See Appendix I for a list of notation),

\[
\max_{OP_i, \theta_i, q_{bi}, \lambda_i} \left\{ \sum_{i=0}^{I} \pi_i^* (cOP_iQ - (\theta_i - 1)(P_i - OP_i)Q) \right\}, \quad \text{s.t.}
\]

\[
(3) \quad P_i = \bar{P} - (I - i)\alpha + m \left( 1 - \frac{1}{\bar{\theta}_i} \right), \quad i = 0, 1, \ldots, I,
\]

\[
(4) \quad \text{(IC1)} \quad \sum_{i=1}^{I} \pi_{i-1}^* (P_i - OP_i) q_{bi,i} \geq \sum_{i=1}^{I} \pi_{i-1}^* (\alpha + P_{i-1} - OP_{i-1}) q_{bi-1,i-1},
\]

\[
(5) \quad \text{(IC2)} \quad \sum_{i=0}^{I} \pi_i^* (P_i - OP_i) q_{bi,i} \lambda_i \geq \sum_{i=0}^{I} \pi_i^* (P_{i+1} - \alpha - OP_{i+1}) q_{bi,i+1},
\]

\[
(6) \quad \text{(IR)} \quad OP_i \leq P_i, \quad i = 0, 1, \ldots, I,
\]

\[
(7) \quad \text{(Q1)} \quad \frac{1}{L} \leq q_{bi,i} \leq 1, \quad i = 0, 1, \ldots, I,
\]

\[
(8) \quad \text{(Q2)} \quad \frac{1}{L} \leq q_{bi,i} \leq 1, \quad i = 0, 1, \ldots, I,
\]

\[
(9) \quad \text{(Q3)} \quad q_{bi,i} + (I - i)q_{bi,i}\lambda_i = \theta_i Q, \quad i = 0, 1, \ldots, I,
\]

\[
(10) \quad \text{(Q4)} \quad 0 \leq \lambda_i \leq 1, \quad i = 0, 1, \ldots, I.
\]

Equation (4) is the incentive compatibility constraint for an investor with a good signal, denoted as (IC1). The probability of having \( i \) good signals conditional on one informed investor already having a good signal is

\[
\pi_{i-1}^* = \left( \frac{I - 1}{I - i} \right)^{i-1} (1 - p)^{I-i}.
\]
Investors can benefit from the issue by obtaining potentially underpriced shares. So, conditioning on a good signal, an informed investor’s expected gain in state $i$ by reporting the good signal is then $\pi^{+}_i (P_i - OP_i)q_{g,i}$. If he lies and reports a bad signal, the offer price will then be $OP_{i-1}$. Meanwhile, the market price will be $P - (I - i)\alpha + m(1 - 1/\theta_{i-1}) = P_{i-1} + \alpha$. Note that the probability of state $i$ occurring will remain the same, so the expected gain by falsely reporting a bad signal is then $\pi^{+}_i (\alpha + P_{i-1} - OP_{i-1})q_{b,i}$. The total expected gain by telling the truth should be no less than the total expected gain from lying. This gives the incentive compatibility constraint for an investor with a good signal as in equation (4). The incentive constraint for an investor with a bad signal, denoted as (IC2) and given by equation (5), can be similarly derived.

Individual rationality requires that risk-neutral investors only accept shares allocated to them if they expect non-negative returns. This gives the individual rationality constraint, denoted as (IR), as in equation (6).

Besides the IR and the IC constraints, four quantitative constraints are also required on the number of shares allocated to investors such that all the shares, including possibly oversold shares, are allocated in the premarket, and the number of shares allocated to each investor is in the range $[L, I]$. These constraints are stated as constraints (Q1)—(Q4).

Before I present the solutions, I want to compare Program PATH to the problem in Benveniste and Spindt (1989). Benveniste and Spindt solve the problem with no overallocation. Their major result is indicated in Figure 1. In Figure 1, $i^*$ is defined as the minimum number of investors with good signals that can take the whole issue.

Note that $i^*$ is defined as the minimum number of investors with good signals that can take the whole issue. For $i \geq i^*$, shares will be allocated to investors with good signals, and underpricing may be present. For $i < i^*$, some shares will be allocated to investors with bad signals, and no underpricing is available.

Note that in Benveniste and Spindt, since no overallocation is involved, the objective function for the underwriter can be reduced to the minimization of the expected amount of money left on the table. This in turn requires giving zero underpricing benefits to investors with bad signals, and to also minimize their allocation if possible. This is how $i^*$ is chosen. However, a careful inspection of the incentive constraint for investors with good signals, which is binding at the optimum, indicates that only total expected underpricing benefits are minimized, and that how the underpricing is distributed is still unknown. In other words, although one knows that underpricing will be present for $i \geq i^*$, one does not know the expected underpricing in each state. To demonstrate the optimization
problem without overselling. I provide detailed numerical examples in Appendix III. The base case corresponds to the Benveniste and Spindt solution.

As will become clear, the search for the optimal overallocation in each state forces one to explicitly specify the underpricing in each state. This makes it impossible to transform the objective function in Program PATH to minimizing the total expected underpricing. In this paper, I rely on intuition, and transform the problem step-by-step into a simpler format until I solve it. This approach also obtains some insights into the IPO allocation process.

B. Allocation and Overselling when Demand Is Strong

IPOs are often oversubscribed, and during the allocation process the underwriter favors investors who demonstrate strong interest in the issue. The model captures this aspect—investors who report good signals have priority in the share allocation. I prove this in Proposition 1 (see Appendix II for proof).

Proposition 1. Investors who report good signals have priority in the premarket share allocation. That is, if \( q_{b,i} \lambda_i > 0 \), we must have \( q_{g,i} = 1 \).

Proposition 1 holds for all \( i \). The intuition behind it is simple. Underpricing is used as an incentive to induce a truthful report from investors with good signals. It is always efficient if the underwriter can reduce the allocation to investors with bad signals.

What happens if there are sufficient good signals? (I will make it clear what I mean by sufficient.) As is typical in the information acquisition framework, the underwriter underprices the IPO if demand is strong. For tractability, two further assumptions are needed. First, underpricing, if any, is non-decreasing in \( i \). Second, underpricing in each state, if not zero, is bounded both above and below. Furthermore, there exists an \( i^{**} \geq i^* \) such that, for any \( i \geq i^{**} \), the amount of underpricing \( (P_i - OP_i) \geq cm/(1 - c) \). The following proposition can then be proved.

Proposition 2. The underwriter will not oversell the issue when there is strong demand, and will allocate all shares to investors who report good signals. That is, for \( i \geq i^{**} \), \( \theta_i = 1 \), \( q_{g,i} = Q/i \), and \( q_{b,i} \lambda_i = 0 \).

Proposition 2 gives the overselling and allocation schedule the underwriter uses when demand is strong \( (i \geq i^{**}) \). When demand is strong, the underwriter does not oversell because the losses of buying back underpriced shares exceed the benefits from an increased market price.

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8This assumption is reasonable. Notice that an increase in \( i \) decreases the number of shares allocated to each investor with a good signal. To smooth the gain for the informed investors with good signals, the underwriter should increase the underpricing accordingly. Empirical evidence about IPO underpricing, such as Bradley and Jordan (2002), is consistent with this assumption.

9This minimum underpricing is assumed for each state because it is needed in proving Proposition 2.

10For \( i < i^* \), since the underwriter has to allocate shares to investors with bad signals, it is always suboptimal to underprice in these states as I will prove later in Proposition 3. So it is safe to assume that the cutoff point, \( i^{**} \), is no less than \( i^* \).
C. Allocation and Overselling when Demand is Weak

When demand is weak \((i < i^{**})\), the issue is not underpriced, and the underwriter first allocates shares to investors reporting good signals. This is Proposition 3. The underwriter should concentrate underpricing in good states where investors reporting bad signals get nothing. This is optimal because the underwriter gains nothing from giving underpriced shares to investors with bad signals.

**Proposition 3.** There is no underpricing when demand is weak. The underwriter first allocates shares to investors with good signals, and gives the rest to investors with bad signals. That is, for all \(i < i^{**}\), \(P_i = OP_i\), \(q_{g,i} = \min(I, \theta_i Q/i)\), and \(q_{b,i} \lambda_i = \max(0, (\theta_i Q - i\bar{I})/(I - i))\).

It is important to note that the allocation of extra shares to investors with good signals does not cost the underwriter, while the allocation of extra shares to investors with bad signals does. This is because the allocation to investors with bad signals determines the required underpricing.\(^{11}\) Separate the total overallocation into two parts—the allocation of extra shares to investors with good signals and the allocation of extra shares to investors with bad signals. Denote the overallocation to investors with bad signals plus the base allocation of \(Q\) shares as \(\theta'_i\), where \(\theta'_i \geq 1\). Note that \(\theta'_i = 1\) means that there is no overselling and the underwriter just allocates \(Q\) shares. Denote the overallocation to investors with good signals as \(\theta_i\), where \(\theta_i \geq 0\).

For \(i < i^\ast\), since there are not enough investors with good signals and each of them is already allocated \(I\) shares, if the underwriter oversells the issue, all oversold shares are allocated to investors with bad signals. That is, \(\theta'_{i} = 0\) for \(i < i^\ast\). For \(i^\ast \leq i < i^{**}\), each investor with a good signal only obtains an allocation of \(Q/i\), which is less than \(I\). The underwriter can then oversell a proportion \(i(I - Q/i)/Q\) to investors with good signals. That is, \(\theta'_i = (I - Q/i)/Q\) for \(i^\ast \leq i < i^{**}\). This is shown in Figure 2. Note that for \(i \geq i^{**}\), no overallocation is involved as in Proposition 2.

The preselling scalar can then be rewritten as

\[
\theta_i = \begin{cases} 
\theta'_i & i < i^\ast \\
\theta'_i + \theta'_i & i^\ast \leq i < i^{**}
\end{cases}
\]

Note that \(\theta'_i\) now becomes the new choice variable because the underwriter will always oversell the \(\theta_j\) part whenever possible. Still, it is not an easy task to find the optimal \(\theta'_i\) and \(i^{**}\). The difficulty is that \(i^{**}\) is discrete, and \(i^{**}\) depends on \(\theta'_i\). To get an approximate solution for \(\theta'_i\), I need to separate the interaction between \(i^{**}\) and \(\theta'_i\).

The underwriter will push \(i^{**}\) as high as possible to leave more room for the \(\theta_j\) overselling (the middle part in Figure 2). Consequently, the underpricing in each state \(i \geq i^{**}\) must have approached its upper limit. If there is an increase in \(\theta'_i\), the underwriter has to lower \(i^{**}\) to incorporate the increased total underpricing. Everything else equal, the reduction of \(i^{**}\) will lower the value of the

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\(^{11}\) This is essentially from the (IC1) constraint. The allocation for an investor with a bad signal determines the required underpricing (the left-hand side) because the gain for an investor with a good signal to lie (the right-hand side) is the product of \(\alpha\) and the allocation for an investor with a bad signal.
FIGURE 2
Underpricing and Share Allocation with Possible Overallocation

$$q_{i,j} = \max \left( 0, \frac{\theta Q - j}{1 - j} \right)$$

No Underpricing

$$q_{i,j} = \frac{Q}{\mu} \cdot q_{i,j} = 0$$

Underpricing

Objective function because this reduces free overallocation. I call this interaction the $i^{**}$ effect. For an increase of $\theta_i^i$ starting from one in state $i$, the $i^{**}$ effect depends on the increase in underpricing in state $i$ (due to the increase in $\theta_i^i$), which is $\beta c (\theta_i^i)(1 - p) \pi_i(\theta_i^i - 1) Q$. It also depends on the original cutoff point $i^{**}$, and probabilities such as $\pi_i^{r_{i^{**}}}$ (and all of these depend on all the parameters in the setup). Without detailed information about the underpricing structure, the magnitude of the $i^{**}$ effect cannot be specified exactly. As a crude approximation, I assume that the $i^{**}$ effect equals $\beta c (\theta_i^i)(1 - p) \pi_i(\theta_i^i - 1) Q$ in state $i$. That is, I assume that the increase in underpricing determines the basic magnitude of the $i^{**}$ effect, and that $\beta > 0$ represents all other factors that are related to the $i^{**}$ effect.

After incorporating the $i^{**}$ effect into the objective function, $i^{**}$ can be treated as fixed while searching for the optimal $\theta_i^i$. The following proposition can then be proved.

Proposition 4. When demand is weak, the underwriter should at least oversell the issue to such a degree that all investors who report good signals get the maximum allocation. More specifically, for $i < i^{**}$, the underwriter should use the following overselling schedule:

i) For $i < i^{**}$, presell the issue up to

$$\sqrt{\frac{m(1 - p)}{pc(1 + \beta)}} \times Q$$

if

$$\sqrt{\frac{m(1 - p)}{pc(1 + \beta)}} > 1,$$

and only allocate $Q$ otherwise;

ii) For $i^{**} \leq i < i^{**}$, presell the issue up to

$$\sqrt{\frac{m(1 - p)}{pc(1 + \beta)}} \times Q$$

if

$$\sqrt{\frac{m(1 - p)}{pc(1 + \beta)}} > (1 + \theta_i^i),$$
and presell \((1 + \theta_i)Q\) otherwise.

Propositions 2 and 4 together complete the picture regarding the underwriter’s overselling behavior in the premarket. While the underwriter will not oversell strong IPOs \((i \geq i^{**})\), it may oversell weak ones. When an IPO is weak \((i < i^{**})\), there is no underpricing. If the oversold shares are only allocated to investors with good signals, the underwriter oversells these shares since this does not incur further underpricing. This is why the overallocation is still \(\theta_i > 0\) for \(i' \leq i < i^{**}\) even if \(\sqrt{m(1 - p)/(p\alpha(1 + \beta))}\) is not big enough to warrant more overselling.

The underwriter faces a trade-off when oversold shares are allocated to investors with bad signals. On the one hand, increasing the number of shares oversold increases the market price and hence the offer price. On the other hand, giving a larger allocation to investors with bad signals creates more incentive for investors with good signals to lie. This incurs more underpricing when demand is strong. This trade-off implies a positive relationship between the number of shares oversold and the effect of overselling on the price, \(m\). Similarly, there is a negative relationship between overselling and the unit incentive for lying about the signal, \(\alpha\). This is the intuition behind \(\partial\theta'_i/\partial m > 0\) and \(\partial\theta'_i/\partial \alpha < 0\). Notice that increasing \(\theta'_i\) also has the \(i^{**}\) effect—the underwriter has to lower the cutoff point, \(i^{**}\), to incorporate the increased underpricing due to an increase in \(\theta'_i\). The stronger the \(i^{**}\) effect, the less the underwriter should oversell. The \(i^{**}\) effect is positively related to \(\beta\), which measures the reduction of \(i^{**}\) for the unit increase of underpricing due to overselling. That is why \(\partial\theta'_i/\partial \beta < 0\).

D. Concentration of Underpricing

The underwriter should always push the underpricing to the upper limit in all states \(i \geq i^{**}\). Notice that, with \(\theta_i\) determined as in Proposition 4, the complete allocation schedule is determined. So is the total expected underpricing. By pushing the underpricing, \(P_i - OP_i\), for all \(i \geq i^{**}\) to the upper limit, the number of good signals that satisfies the (IC1) constraint is then the optimal value for \(i^{**}\). This is summarized as Proposition 5.

Proposition 5. After determining the overselling in states \(i < i^{**}\), the underwriter should set \(i^{**}\) as high as allowed by the upper limit of underpricing and the total expected underpricing. That is, the underwriter should concentrate all expected underpricing (as defined in (IC1)) in states \(i \geq i^{**}\).

Proposition 5 points to an interesting feature of the model: the concentration of underpricing in states with strong demand. For presentation purposes, define three types of IPOs: hot IPOs \((i \geq i^{**})\), mediocre IPOs \((i^* \leq i < i^{**})\), and cold IPO \((i < i^*)\). The concentration of underpricing occurs because the underwriter faces incentives to leave more room to oversell in the middle \((i^* \leq i < i^{**})\) (see Figure 2). Note that in Benveniste and Spindt (1989), the underwriter is indifferent in underpricing hot or mediocre IPOs because IPO investors who report bad signals will not receive shares without over-allocation. So in their model both hot and mediocre IPOs are grouped as hot IPOs. Since the IPO literature usually uses the dichotomy of hot and cold IPOs, for presentation purposes, both mediocre
itively related to the allocation to investors with bad signals in cold IPOs. If the underwriter only allocates oversold shares to investors with good signals, it can increase the aftermarket price without increasing underpricing. Hence, any over-allocation that goes to investors with good signals is costless. For mediocre IPOs, investors with good signals will not obtain the maximum allocation. To maximize the costless overallocation, everything else equal, the underwriter will set $i^{**}$ as high as possible by pushing underpricing to its upper limit for hot IPOs. Such concentration of underpricing will generate an extremely skewed distribution of first-day returns. Notice that this occurs only because of the path dependency hypothesis.

E. Summary

Propositions 1 through 5 give the complete pricing and allocation schedule if the underwriter oversells the IPO and such overselling affects the issue’s market price. Figure 2 illustrates how underpricing and allocation change when demand (number of good signals) changes. Two numeric examples, Cases 2 and 3 in Appendix III, are provided to demonstrate the intuition why the underwriter only oversells mediocre and cold IPOs.

Overselling cold IPOs results in greater aftermarket demand, which allows cold IPOs to be sold at a higher offer price. The cost of overselling cold IPOs is that hot IPOs are further underpriced to maintain truthful information revelation. Can the underwriter and the issuing firm benefit from the overselling? The answer is yes because the increased offer price will offset the increased underpricing. I summarize this as Proposition 6.

**Proposition 6.** Under the path dependency hypothesis, both the underwriter and the issuing firm benefit from the premarket overallocation. The expected gross proceeds increase due to the premarket overallocation.

V. Discussion and Empirical Implications

In this paper, I argue that price path dependency, not price support, is the motivation for IPO overselling. The information acquisition framework adopted in this paper provides a particular interaction between underpricing and allocation. This sets an upper limit of overselling in my model. I want to emphasize that path dependency is the key assumption in the model. The adoption of the information acquisition framework makes my results comparable to existing theoretical results. But such a framework is not crucial.

Other explanations and conjectures about overselling weak IPOs exist. Wilhelm (1999) conjectures that overselling is used by the underwriter to create aftermarket buying power to support the stock’s price. However, substitution is a major issue here. The underwriter’s buying power may just be a substitute for investors’ buying power, since potential buyers would buy shares in the aftermarket if they had not been allocated shares. Wilhelm does not explain why this would
not be an issue. The path dependency argument suggests that the main reason for the premarket overselling is not to create aftermarket buying power. Overselling results in greater aftermarket demand, and this does not come at the expense of removing aftermarket buying power.

Despite the substitution issue, overselling gives the underwriter the freedom to time its short position covering to balance the flow supply and demand. While flippers create selling pressure during the first few days’ trading, buyers may not step in when they are most needed. By overselling the issue and taking a short position, the underwriter can time the buying and step in when it is most needed. In this sense, overselling can be viewed as a way to support the market to offset temporary supply-demand imbalances.

Fishe (2002) argues that underwriters oversell weak IPOs for larger profits, and that price support, if any, is only a side product. He argues that flippers participate in the premarket and their artificial demand drives up the supply-demand equilibrium price. When they flip their shares, the aftermarket price will go down. The underwriter takes a short position to take advantage of this. The major concern about Fishe’s setup is the true buyers’ (relative to flippers) response to the artificial demand. The true buyers will demand more “underpricing” due to the existence of flippers. This will eliminate any chance from which the underwriter profits.

Extant empirical evidence is consistent with the major predictions of my model. Most existing theoretical explanations for price support (Chowdhry and Nanda (1996) and Benveniste, Busaba, and Wilhelm (1996)) take stabilizing bids as the method of price support, and assume that underwriters do not oversell shares in the premarket. Aggarwal (2000) shows that almost all IPOs that need price support are oversold by more than the 15% overallotment option, and that stabilizing bids are not used at all. Furthermore, the existing literature also assumes that price support is costly to underwriters. However, both Aggarwal (2000) and Ellis, Michaely, and O’Hara (2000) find that aftermarket short covering is not costly to the underwriter. These findings—only weak IPOs are oversold and overselling is not costly to the underwriter—are consistent with my model. The model also predicts a skewed distribution of the IPO first-day return due to the concentration of underpricing. Numerous studies have documented this pattern.

My model also provides further testable empirical implications. The model suggests that the naked short position is positively correlated with the effect overselling has on the stock price. This overselling effect should be bigger for smaller and less well-known IPOs. Consequently, I would expect larger proportional overselling for those IPOs. However, the amount of overselling is also negatively correlated with the cost of information. It is plausible that information gathering is more valuable for smaller and less well-known IPOs. This would result in less overselling for these IPOs. In other words, the amount of overselling depends on both the cost of information and the effect of overselling on price. It is an empirical question how these factors interact to affect the amount of overselling. Furthermore, due to the free overselling for mediocre IPOs with moderate demand, more frequent small overallocations may be seen for those IPOs. This

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13Benveniste, Busaba, and Wilhelm (1996) also point out that weak IPOs are oversold, but they do not provide empirical evidence.
may help to distinguish my theory from the aftermarket buying power creation argument proposed by Wilhelm (1999).

VI. Conclusion

The academic literature on price support has long assumed that underwriters use stabilizing bids, that is, a commitment to buying back shares at the offer price if needed, to support IPOs in the aftermarket. However, the evidence provided by Aggarwal (2000) gives a totally different picture—if the IPO faces weak demand, where stabilizing bids should presumably be needed, underwriters overallocate the stock and take a naked short position. In other words, a major component of price support (as defined by Aggarwal) is based on ex ante allocation strategies, rather than ex post purchasing decisions. This paper proposes an equilibrium model to explain this overselling phenomenon.

I argue that the stock price of an IPO is path dependent, and overallocation can result in greater aftermarket demand and increase the market price. Under an information acquisition framework first proposed by Benveniste and Spindt (1989), hot IPOs are underpriced to provide incentives for truthful information revelation. The underpricing of hot IPOs prevents overselling because it is too costly to buy oversold shares at a higher price.

Cold IPOs are fairly priced. Overallocating cold IPOs results in a higher aftermarket price. However, overallocating cold IPOs requires further underpricing of hot IPOs, because increased allocations for investors reporting a bad signal require greater incentives to induce truthful information revelation. The overallocation strategy increases the unconditional expected offer price when the effect of an increased aftermarket demand on cold IPOs more than offsets the effect of greater required underpricing of hot IPOs.

Price path dependency is the key assumption that drives my results regarding overallocation. Essentially I argue that underwriters use the road show and bookbuilding not only to collect information and price the IPO, but also as an intensive marketing campaign to promote the stock. But I want to point out that it is controversial to assume that the price of a security can be affected by marketing tactics. Further research on marketing of IPOs is needed.

Appendix I. Notation

$I$: total number of investors.
$Q$: total number of shares offered, including shares in the overallotment option.
$\bar{I}$: minimum (maximum) number of shares an investor can take.
$p$: probability for a good signal, and $1 - p$ is the probability for a bad signal.
$i$: number of good signals, $i \in \{0, I\}$. State $i$ refers to the situation of $i$ good signals.
$i^*$: the minimum number of investors with good signals that can take the whole issue.
$i^{**}$: the largest number of good signals such that the total underpricing for all states $i \geq i^{**}$ is sufficient to induce truth-telling.
$\pi_i$: probability of having $i$ good signals,

$$\pi_i = \binom{I}{i} p^i (1 - p)^{I-i}. $$
Obviously (IC1) will still hold, and the objective function is increased. This proves that
\( q_{b,i} \) will not be binding at the optimum), and the left-hand side of (A-1) increases by
after the re-allocation, all other constraints will still hold (the second IC constraint will
so, we can pick any
as
\( q_g \) and increase
\( \text{the effect of overselling on the stock price.} \)
\[ \theta_i: \text{overselling scalar, } \theta_i \geq 1. \]
\[ q_{b,i}(q_{b,i}): \text{allocation for an investor reporting a good (bad) signal in state } i. \]
\[ q_{b,i} \in [0,1]. \]
\[ \lambda_i: \text{probability for an investor reporting a bad signal to be allocated } q_{b,i} \text{ shares in state } i. \]
\[ P_i: \text{market price in state } i. \]
\[ OP_i: \text{offer price in state } i. \]
\[ P: \text{maximum price the stock can reach when all signals are good.} \]
\[ \alpha: \text{the effect of information—one good signal increases the stock price by } \alpha. \]
\[ m: \text{the effect of overselling on the stock price.} \]

Appendix II. Proofs of Propositions

Proof of Proposition 1. This can be proved by contradiction. Suppose for a given \( i' \), that
\( q_{b,i'} \lambda_{i'} > 0 \) and \( q_{b,i'} < 1 \) at the optimum. Notice that the (IC1) constraint can be rewritten as
\[ (A-1) \]
\[ \pi_{i'}(P_{i'} - OP_{i'})q_{b,i'} + \sum_{i \neq i'} \pi_{i'}(P_i - OP_i)q_{b,i} \geq \]
\[ \pi_{i'}(\alpha + P_{i'} - OP_{i'})q_{b,i'} \lambda_{i'} \]
\[ + \sum_{i \neq i'} \pi_{i'}(\alpha + P_{i'} - OP_{i'})q_{b,i} \lambda_{i}. \]
Increase \( q_{b,i'} \) by \( \Delta \) (a small number), and decrease \( q_{b,i'} \lambda_{i'} \) by \( (i/(1-i)) \Delta. \) Notice that, after the re-allocation, all other constraints will still hold (the second IC constraint will not be binding at the optimum), and the left-hand side of (A-1) increases by \( \pi_{i'}(P_{i'} - OP_{i'}) \Delta \geq 0, \) while the right-hand side decreases by \( \pi_{i'}(\alpha + P_{i'} - OP_{i'}) \Delta > 0. \) So, we can pick any \( i \) for which \( P_i > OP_i \) and increase \( OP_i \) by a very small number \( \Delta'. \) Obviously (IC1) will still hold, and the objective function is increased. This proves that
\( q_{b,i'} \lambda_{i'} > 0 \) and \( q_{b,i'} < 1 \) cannot be an optimum.

Proof of Proposition 2. Substituting equation (1) into the objective function, and dropping the constant term, the objective function can be rewritten as
\[ (A-2) \]
\[ \sum_{i < i^*} \pi_{i'} cm \left[ 1 - \frac{1}{\theta_i} \right] Q - \sum_{i < i^*} \pi_{i'}(\alpha_i + 1)(P_i - OP_i)Q \]
\[ + \sum_{i \geq i^*} \pi_{i'} cm \left[ 1 - \frac{1}{\theta_i} \right] Q - \sum_{i \geq i^*} \pi_{i'}(\alpha_i + 1)(P_i - OP_i)Q. \]
Keep the allocation and pricing schedule fixed for all \( i < i^*. \) Let \( H = \sum_{i > i^*} \pi_{i'} cm[1 - (1/\theta_i)]Q - \sum_{i \geq i^*} \pi_{i'}(\alpha_i + 1)(P_i - OP_i)Q. \) Note that \( \partial H/\partial \theta_i = \pi_{i'} cm(1/\theta_i^2)Q - \pi_{i'}(P_i - OP_i)Q < 0 \) if we keep the underpricing constant. So, if \( q_{b,i} \lambda_{i} > 0, \) we can make \( q_{b,i} \lambda_{i} = 0 \)
to reduce $\theta_i$, and the objective function will be increased. Notice that (IC1) still holds after changing $q_{bi}\lambda_i > 0$ to $q_{bi}\lambda_i = 0$. So, we must have $q_{bi}\lambda_i = 0$ at the optimum.

Given that $q_{bi}\lambda_i = 0$ for all $i \geq i^*$, we can rewrite the (IC1) constraint as

\begin{equation}
(A-3) \quad \sum_{i \geq i^*} \pi_{i-1}(P_i - OP_i) q_{bi,i} \geq \sum_{i < i^*+1} \pi_{i-1}(\alpha + P_{i-1} - OP_{i-1}) q_{bi,i-1}\lambda_{i-1} - \sum_{i < i^*} \pi_{i-1}(P_i - OP_i) q_{bi,i}.
\end{equation}

Notice that the right-hand side of (A-3) is a constant because the pricing and allocation schedule are fixed at the original optimum for all $i < i^*$. Now we can prove that $\theta_i = 1$ for all $i \geq i^*$. Notice that $H = \sum_{i \geq i^*} \pi_{i-1}(p/l)i\theta_i[1 - (1/\theta_i)]Q - \sum_{i < i^*+1} \pi_{i-1}((c + \theta_i - 1)/\theta_i)P_i(OP_i)q_{bi,i}$. When we change $\theta_i$ and hence $q_{bi,i}$ for all $i \geq i^*$, let us change the underpricing $P_i - OP_i$ accordingly so that $(P_i - OP_i)q_{bi,i}$ is unchanged. So, from equation (A-3) we know that the (IC1) constraint still holds. Given any $\theta_i > 1$ where $i \geq i^*$, we have $\partial H/\partial \theta_i < 0$. So, we can reduce the overselling until $\theta_i = 1$, and the objective will increase while all constraints still hold. So $\theta_i = 1$ is the optimum for $i \geq i^*$. 

**Proof of Proposition 3.** By substituting equations (1) and (9) in the text into the objective function, and dropping the constant term, we can rewrite the objective function as

\begin{equation}
(A-4) \quad \sum_{i=0}^{t} \pi_{i} c m \left[ 1 - \frac{1}{\theta_i} \right] Q - \left\{ \sum_{i=1}^{t} \frac{c + \theta_i - 1}{\theta_i} \pi_{i-1} p l (P_i - OP_i) q_{bi,i} + \sum_{i=1}^{t} \frac{c + \theta_i - 1}{\theta_i} \pi_{i-1} (1 - p) l (P_{i-1} - OP_{i-1}) q_{bi,i-1}\lambda_{i-1} \right\}.
\end{equation}

For any $i < i^*$, if $P_i > OP_i$, we can always increase $OP_i$ such that $P_i = OP_i$, and meanwhile increase the underpricing in states $i \geq i^*$ such that the (IC1) constraint, which will be binding at the optimum, still holds. The change in the objective function is then greater than or equal to $-\sum_{i < i^*+1} \pi_{i-1}(p/l)i\theta_i[1 - (1/\theta_i)]Q - \sum_{i < i^*+1} \pi_{i-1}((c + \theta_i - 1)/\theta_i)P_i(OP_i)q_{bi,i} + \pi_{i-1}((c + \theta_i - 1)/\theta_i)P_i(OP_i)q_{bi,i}$, and hence

\begin{equation}
\sum_{i < i^*+1} \pi_{i-1}(p/l)i\theta_i[1 - (1/\theta_i)]Q - \sum_{i < i^*+1} \pi_{i-1}((c + \theta_i - 1)/\theta_i)P_i(OP_i)q_{bi,i} + \pi_{i-1}((c + \theta_i - 1)/\theta_i)P_i(OP_i)q_{bi,i} \geq 0.
\end{equation}

This proves that $P_i = OP_i$ is optimal for any $i < i^*$.

The allocation schedule stated in Proposition 3 follows from Proposition 1. 

**Proof of Proposition 4.** Using Propositions 2 and 3, the objective function in equation (A-2) can be re-stated as $\sum_{i < i^*+1} \pi_{i} c m [1 - (1/\theta_i)]Q - \sum_{i < i^*+1} \pi_{i} c (P_i - OP_i)Q$. Notice that (IC1) is binding at the optimum, the objective can be further rewritten as $\sum_{i < i^*+1} \pi_{i} c m [1 - (1/\theta_i)]Q - \sum_{i < i^*+1} \pi_{i} c (P_i - OP_i)Q_{bi,i}$, where $Q_{bi,i} = (1 - i)q_{bi,i}\lambda_i$ is the total number of shares allocated to investors with bad signals.

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14By keeping $(P_i - OP_i)q_{bi,i}$ constant, we can ignore this term when we differentiate $H$ with respect to $\theta_i$.

15The changes of underpricing due to allocations to investors with bad signals are not included. From equation (A-4) and the constraint (IC1), it is clear that including these changes will make the objective even greater.
Note that \( Q_{b,i} = \max\{0, \theta_i Q - i \} \), and after separating overallocation into two parts as defined in equation (11) in the text, we can then rewrite the objective function as

\[
(A-5) \quad \sum_{i < i^*} \pi_i c \left[ m \left( 1 - \frac{1}{\theta_i} \right) Q - \frac{\rho_i}{1 - p} \ (\theta_i Q - i \times i) \right] \\
+ \sum_{i^* \leq i < \bar{i}^*} \pi_i c \left[ m \left( 1 - \frac{1}{\theta_i + \theta_i'} \right) Q - \frac{\rho_i}{1 - p} \ ((\theta_i + \theta_i') Q - i \times i) \right].
\]

Incorporating the \( i^* \) effect into the objective function, and we then have

\[
(A-6) \quad \sum_{i < i^*} \pi_i c \left[ m \left( 1 - \frac{1}{\theta_i} \right) Q - \frac{\rho_i}{1 - p} \ (\theta_i Q - i \times i) - \beta \frac{\rho_i}{1 - p} \ (\theta_i - 1) \right] \\
+ \sum_{i^* \leq i < \bar{i}^*} \pi_i c \left[ m \left( 1 - \frac{1}{\theta_i + \theta_i'} \right) Q - \frac{\rho_i}{1 - p} \ ((\theta_i + \theta_i') Q - i \times i) - \beta \frac{\rho_i}{1 - p} \ (\theta_i - 1) \right].
\]

Since \( i^* \) is fixed when we search for the optimal \( \theta_i' \) after the transformation, we can then differentiate equation (A-6) with respect to \( \theta_i' \) and take the first order condition. Solving the FOC would give us the results as in Proposition 4. Note that the second order condition is satisfied because \( \theta_i' \geq 1 \).

**Proof of Proposition 6.** For the no-overallocation scenario, we can set \( \theta_i = 1 \) for all \( i \) in Program \text{PATH} to get the solutions. Note that this is simply the starting point of the optimization problem with overallocation. Then it is obvious that if the underwriter chooses any \( \theta_i > 1 \), the objective function must be greater than the no-overallocation scenario.

**Appendix III. Numerical Examples**

Three numerical examples are constructed: the Base Case, the Overselling of Cold IPOs, and the Overselling of All IPOs. In all three examples, the total number of shares offered is 100, the probability for an investor to receive a good signal is 0.3, and the probability for an investor to receive a bad signal is 0.7. There are a total of 10 investors. I assume that the IPO can only take two values: hot with a market price of 20, and cold with a market price of 10 (without overallocation). I also assume that a total of six or more good signals are needed so that the IPO will be hot. Otherwise, it would be a cold IPO. Note that this is a simplification of the model in the paper. In the paper, each signal will affect the market price, and here the price only changes when the number of good signals drops from six to five. However, since each investor independently reports his signal to the underwriter, this simplification will still carry through the basic intuition (and the way the model works).

The maximum allocation each investor can take is assumed to be 18. Note that this number is determined following the way \( i^* \) is determined in the Benveniste and Spindt (1989) model. That is, given this maximum allocation, at least six good signals are needed for those who report good signals to take the whole issue. This is also why the hot IPO is defined as the case where there are six or more good signals. Note that I do not specify the minimum allocation requirement. For investors who report good signals, the minimum allocation is not binding. For investors who report bad signals, since I assume that each investor will get the allocation for investors who report bad signals with a probability less than one, the minimum expected allocation for a single investor who reports a bad signal will be zero.
The IPO prices (both High and Low) and the overallocation are at the top of each case. I assume that the possible overallocation is either 20 or zero. This is again a simplification from the model in the paper. However, given that the overallocation size is the same for different IPOs as shown in Proposition 4, this simplification will not affect the basic intuition of the model.

Below are explanations for each row in the table of each case:

**Actual Number of Gs:** the number of good signals, which could range from zero to 10.

**Probability:** the probability for each state, which is calculated using

\[ \pi_i = \binom{I}{i} p^i (1 - p)^{I-i} \]  
(where \( p = 0.3 \), \( I = 10 \)).

**Allocation for G:** allocation for an investor who reports a good signal.

**Allocation for B:** allocation for an investor who reports a bad signal. As shown in Proposition 1, shares first go to those who report good signals, and whatever is left will be divided among investors who report bad signals.

**Expected Lying Benefits:** the benefits that an investor who has a good signal but reports a bad signal could expect to get. Such benefits equal the product of the following: the probability of the true state in which this investor receives a good signal, the allocation for B in the next state with one less good signal, and the difference between the market price of the true state and the offer price in the next state. Due to the setup, such lying benefits only exist when the true number of signals is six.

**Marginal Underpricing Effect:** the allocation for G in the state multiplied by the probability of the state when the IPO is hot, and zero otherwise (note that I could do this because the cold IPO will be fairly priced). Note that the numbers are small (shown as zero because of rounding) in top states simply because of the small probability. Note that I add up the marginal underpricing effect in each state at the end, and this is the total expected underpricing effect. The total expected underpricing effect is used to calculate the required dollar underpricing. Note that we have Required Underpricing ($) \times Total Underpricing Effect = Total Expected Lying Benefits as required by (IC1) in the model. (So this row just reports some intermediate results that will be used to calculate underpricing.)

**Market Price:** the market price based on the number of good signals. I assume it would be either 20 or 10.

**Offer Price:** the price at which the underwriter will sell the issue. The cold IPO is fairly priced, and the offer price for the hot IPO is simply the market price minus the Required Underpricing (in dollars).

**Proceeds:** the offer price in the state multiplied by the number of shares offered. The number in the last column is the expected proceeds from the issue, including gross spreads.

**Underwriting Discount:** the gross spreads, which is assumed to be 7%.

**Cost of Buying Back Shares:** the difference between offer price and the market price in each state multiplied by the number of shares oversold. The last column is the total expected cost, which is the summation of the product of the cost in each state and the corresponding probability.

**Required Underpricing ($):** dollar underpricing when the IPO is hot, which simply equals total expected lying benefits divided by total underpricing effect (both are reported in the last column in the corresponding row). Please note that the setup of the so-called underpricing effect guarantees that this will be the dollar underpricing in each state when the IPO is hot.

**Underwriting Revenue:** equals underwriting discount (gross spreads) minus cost of buying back shares.

Three cases follow. The first case is the base case, which is simply an illustration of the Benveniste and Spindt model. The second case illustrates what happens if cold
IPOs are oversold, and the third case shows what happens if both cold and hot IPOs are oversold. At the bottom of each case, we can see that underwriting revenue is the highest when only cold IPOs are oversold. The underwriter will not oversell hot IPOs because the cost of buying back underpriced shares is more than the increase in the underwriting discount (note that the total proceeds still increase).

### Case 1

This is the base case. No overallocation occurs. Note that the hot IPO is still underpriced.

<table>
<thead>
<tr>
<th>Case</th>
<th>Base Case</th>
<th>Cold Only Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Actual no. of Gs</td>
<td>0</td>
</tr>
<tr>
<td>Probability</td>
<td>0.0282</td>
<td>0.1211</td>
</tr>
<tr>
<td>Allocation for G</td>
<td>N/A</td>
<td>18.00</td>
</tr>
<tr>
<td>Allocation for B</td>
<td>10.00</td>
<td>9.11</td>
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<tr>
<td>Expected lying benefits</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Marginal underpricing effect</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Market price</td>
<td>10.00</td>
<td>10.00</td>
</tr>
<tr>
<td>Offer price</td>
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<td>10.00</td>
</tr>
<tr>
<td>Proceeds</td>
<td>1000.00</td>
<td>1000.00</td>
</tr>
<tr>
<td>Underwriting discount</td>
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<td>7%</td>
</tr>
<tr>
<td>Cost of buying back shares</td>
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<td>0.00</td>
</tr>
<tr>
<td>Required underpricing ($)</td>
<td>0.97</td>
<td>0.97</td>
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</tbody>
</table>
Case 3

Twenty shares are oversold for all IPOs, cold and hot. The market prices consequently become $22 and $12. That is, there is a $2 increase for both hot and cold IPOs. Note that the cost of buying back oversold shares is not zero anymore, and this actually reduces the underwriter’s revenue. At the bottom the expected proceeds (before the gross spreads) and underwriting revenue are reported for the previous two cases for comparison.

<table>
<thead>
<tr>
<th>All IPOs Case</th>
<th>High 22</th>
<th>Low 12</th>
<th>Overallocation 20 (all IPOs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual no. of Gs</td>
<td>0.0238</td>
<td>0.1211</td>
<td>0.2335</td>
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<tr>
<td>Allocation for G</td>
<td>N/A</td>
<td>18.00</td>
<td>18.00</td>
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<tr>
<td>Allocation for B</td>
<td>12</td>
<td>11.33</td>
<td>10.50</td>
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<tr>
<td>Expected benefits</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Marginal underpricing effect</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Market price</td>
<td>12.00</td>
<td>12.00</td>
<td>12.00</td>
</tr>
<tr>
<td>Offer price</td>
<td>12.00</td>
<td>12.00</td>
<td>12.00</td>
</tr>
<tr>
<td>Proceeds</td>
<td>1200.00</td>
<td>1200.00</td>
<td>1200.00</td>
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<tr>
<td>Underwriting discount</td>
<td>1937.31</td>
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<td>1937.31</td>
</tr>
<tr>
<td>Cost of buying back shares</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

| Required underpricing ($) | 2.63 |
| Underwriting revenue | 83.96 |
| Base case: no overallocation |
| Expected proceeds | 1042.77 |
| Underwriting revenue | 72.99 |
| Cold IPOs only |
| Expected proceeds | 1230.90 |
| Underwriting revenue | 85.88 |

References


