On Regulatory Responses to the Recent Crisis:
An Assessment of the Basel Market Risk Framework and the Volcker Rule

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Banks around the world suffered huge trading losses in the recent crisis. In response, the Basel Committee on Banking Supervision (2011) provides a revised framework to determine the minimum capital requirements for their trading portfolios. Moreover, the Dodd-Frank Wall Street Reform and Consumer Protection Act (2010) imposes certain restrictions on the composition of the trading portfolios of U.S. banks through the so-called Volcker rule. Our paper assesses the effectiveness of the Basel framework and the Volcker rule in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties. We find that the Basel framework is ineffective in preventing banks from doing so, but that the Volcker rule partially mitigates this ineffectiveness. We also suggest alternatives to the Basel framework and discuss the impact of the Volcker rule if any one of them is adopted.

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1. Introduction

Banks around the world suffered huge trading losses in the recent crisis (hereafter, ‘crisis’).\(^1\) These losses suggest that such banks take substantive tail risk within their trading portfolios. In response to the crisis, the Basel Committee on Banking Supervision (2011) provides a revised framework (hereafter, ‘Basel framework’) for attempting to control the amount of tail risk that these banks take in their trading portfolios.\(^2\) In this framework, the minimum capital requirement associated with a bank’s trading portfolio depends on its VaR and stressed VaR, but the bank must also have a stress testing program in place (see Basel Committee on Banking Supervision (2011, pp. 15–16)).\(^3\)

Also in response to the crisis, the Dodd-Frank Wall Street Reform and Consumer Protection Act (2010, hereafter ‘Dodd-Frank Act’) imposes restrictions on the composition of the trading portfolios of U.S. banks. Since these restrictions were originally proposed by former Chair of the U.S. Federal Reserve Paul Volcker, they are often referred to as the ‘Volcker rule.’ This rule prohibits such banks from engaging in proprietary trading of some assets (e.g., stocks) while permitting them to do so in other assets (e.g., U.S. government and agency obligations).\(^4\)

These regulatory responses to the crisis raise two questions. First, is the Basel framework effective in preventing banks from taking substantive tail risk in their trading portfolios

\(^1\) For a discussion of the causes of the crisis, see Claessens, Demirgüç-Kunt, and Moshirian (2009), Kane (2009), Caprio, Demirgüç-Kunt, and Kane (2010), Dewatripont, Rochet, and Tirole (2010, Ch. 2), Levine (2010a), and Gorton and Metrick (2012).

\(^2\) The implementation of this framework is now complete in many (but not in all) countries; see Basel Committee on Banking Supervision (2012a, p. 6).

\(^3\) In the original Basel framework for trading portfolios, the minimum capital requirement associated to a bank’s trading portfolio solely depends on VaR, but again the bank must also have a stress testing program in place; see Basel Committee on Banking Supervision (1996a, pp. 45–46).

\(^4\) For details on the trading activities that are either prohibited or permitted by the Volcker rule, see Dodd-Frank Act (2010, Section 619). In November 2011, regulators requested comments from the public on their proposed rule to implement the Volcker rule; see Federal Register (2011). These comments are available at: <www.sec.gov/comments/s7-41-11/s74111.shtml>. Regulators are currently working on the final rule that will implement the Volcker rule; see: <www.federalreserve.gov/newsevents/reform_milestones_proposed_rules_being_finalized.htm>. Banks will have to comply with the Volcker rule by July 21, 2014; see: <www.sec.gov/news/press/2012/2012-70-policystatement.pdf>. Volcker (2012, p. 132) suggests that the recent trading losses of European banks motivates the imposition of restrictions on the proprietary trading activities of non-U.S. banks.
without capital requirement penalties? Second, to what extent does the Volcker rule improve the effectiveness (if at all) of the Basel framework? Our paper attempts to shed light on these two questions.

In seeking answers to such questions, we consider a risk management system based on the Basel framework. This system involves a constraint limiting the sum of VaR and stressed VaR as well as constraints limiting losses in stress testing events. The former constraint is motivated by the fact that the Basel framework sets the minimum capital requirement associated with a bank’s trading portfolio to be the sum of two terms involving VaR and stressed VaR, whereas the latter constraints are motivated by the fact that the framework requires banks to have a stress testing program in place.

Since Conditional Value-at-Risk (CVaR) has advantages over VaR, several researchers recommend using CVaR as a measure of tail risk. Accordingly, we follow this recommendation. Specifically, we examine whether the aforementioned constraints preclude the selection of all portfolios with substantive efficiency losses relative to the mean-CVaR frontier. Here, a portfolio belongs to the mean-CVaR frontier if there is no portfolio with the same expected return and smaller CVaR. Also, a portfolio’s efficiency loss is the difference between: (1) its CVaR and (2) the CVaR of the portfolio on this frontier with the same expected return. If the constraints preclude all portfolios with substantive efficiency losses from being selected, then they are effective in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties. However, if the constraints allow the selection of such portfolios, then they are ineffective in preventing banks from doing so.

CVaR has two advantages over VaR. First, CVaR considers the size of losses beyond VaR, whereas VaR does not; see, e.g., Alexander, Baptista, and Yan (2012) and references therein. Second, CVaR is sub-additive (i.e., the CVaR of a two-asset portfolio is less than or equal to the sum of the asset CVaRs), but VaR is not; see, e.g., Artzner, Delbaen, Eber, and Heath (1999). However, Garcia, Renault, and Tsafack (2007) find that the cases where VaR is not sub-additive are rare. There is an extensive literature that recognizes the drawbacks of using VaR as a measure of tail risk. However, the literature has yet to compare the effectiveness of jointly using VaR, stressed VaR, and stress testing in accordance to the Basel framework to control tail risk in the cases where the Volcker rule is either absent or present. Our paper fills this gap in the literature.
In examining the effectiveness of the constraints, we consider plausible problems of wealth allocation among various sets of assets. Our base case for assessing the Basel framework involves the following assets: (a) Treasury bills; (b) Treasury bonds with five maturity ranges (1–3, 3–5, 5–10, 10–15, and 15+ years); (c) Agency bonds with the same maturity ranges; and (d) the six size/book-to-market-based Fama-French equity portfolios. For assessing the impact of the Volcker rule, we examine the effect of removing the Fama-French equity portfolios from consideration. Our robustness checks use Treasury and Agency bonds with six (instead of five) maturity ranges, and the ten size-based (instead of the six size/book-to-market-based) Fama-French equity portfolios.

We utilize historical simulation to estimate VaR, stressed VaR, CVaR, and losses in stress testing events given its popularity among banks (see, e.g., Pérignon and Smith (2010)). In our base case, we consider four stress testing events: (i) U.S. stock market crash of 1987; (ii) U.S. interest rate hike of 1994; (iii) terrorist attacks of 2001; and (iv) subprime crisis of 2007–09. For robustness checks, we consider four additional stress testing events: (v) Russian crisis of 1998; (vi) LTCM collapse of 1998; (vii) dot-com slowdown of 2001–02; and (viii) Greek crisis of 2010.

First, we assess the effectiveness of the Basel framework in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties. We find that the use of constraints limiting the sum of VaR and stressed VaR as well as losses in stress testing events allows the selection of portfolios with substantive efficiency losses. This finding suggests that the Basel framework is ineffective.

Second, we assess the extent to which the Volcker rule improves the effectiveness of the Basel framework. In the presence of this rule, we find that portfolios with substantive efficiency losses can still be selected, albeit to a lesser extent than its absence. This finding suggests that the Volcker rule is beneficial in that it partially mitigates the ineffectiveness of
the Basel framework.

In our view, an unqualified claim that the Basel framework and the Volcker rule are effective requires them to be fully effective in simple settings that are consistent with common practice among banks. Accordingly, our base-case findings raise doubts about: (a) the effectiveness of the Basel framework in preventing banks from doing so; and (b) the adequacy of the Volcker rule in fully mitigating the ineffectiveness of the Basel framework. Our robustness checks indicate that these doubts remain even if we increase the complexity of our setting by considering either a larger number of stress testing events, or a larger number of assets, or both.

Next, we discuss alternatives to the Basel framework. Instead of using VaR and stressed VaR to determine the minimum capital requirements for trading portfolios, we suggest the use of either: (1) CVaR; (2) VaRs at multiple confidence levels; or (3) VaR at a confidence level higher than 99%. The adoption of any one of these alternatives would mitigate the ineffectiveness of the Basel framework. To the extent that such alternative frameworks do so, the benefits of the Volcker rule would be smaller.

It is worth emphasizing that the adoption of the Volcker rule has costs. Duffie (2012) and Thakor (2012) point out that it will adversely affect the liquidity of many securities. As a result, they conclude that the Volcker rule will negatively affect businesses by harming their ability to raise capital and increasing their cost of capital. Also, Thakor recognizes that by constraining bank security holdings, this rule will make bank risk management less efficient.

There is an extensive literature on the impact of bank capital regulation on bank risk-taking activities (see, e.g., Bhattacharya, Boot, and Thakor (1998), Barth, Caprio, and

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6 For example, Duffie notes the adverse impact of the Volcker rule on the liquidity of non-U.S. sovereign bonds as well as U.S. corporate bonds and non-agency mortgage-related securities. Also, he argues that this rule will reduce the liquidity of derivative securities that are used to hedge U.S. Treasury and Agency bonds, which in turn will reduce the liquidity of such bonds. See also his interview to The Region available at: <www.minneapolsfed.org/pubs/region/12-06/Region_June_2012_Full_Issue>.
Levine (2008), and Freixas and Rochet (2008, Ch. 9) for a review). A number of papers theoretically show that bank capital regulation might increase bank risk-taking activities (see, e.g., Koehn and Santomero (1980), Kim and Santomero (1988), and Rochet (1992)). In a related paper, Barth, Caprio, and Levine (2004) do not find robust empirical evidence that stringent capital requirements reduce these activities. Other papers explicitly criticize the original Basel framework for trading portfolios (for a description of this framework, see Basel Committee on Banking Supervision (1996a)). For example, Lucas (2001) and Kane (2006) show that it provides incentives for banks to underreport their estimates of minimum capital requirements based on VaR. Additionally, Pritsker (2006) finds that the use of a short window for VaR estimation purposes may result in severely understated minimum capital requirements. Alexander, Baptista, and Yan (2012) find that the minimum capital requirements set by the 1996 Basel framework are more likely to be wiped out by trading losses than the ones set by the 2011 Basel framework.

However, this literature has yet to examine whether the regulatory responses to the crisis embodied in both the Basel framework and the Volcker rule are effective in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties. Our work adds to the literature by suggesting that these responses are not fully effective in doing so. Specifically, we find that banks that comply with the Basel framework and the Volcker rule can take substantive tail risk in their trading portfolios without capital requirement penalties. Furthermore, we find that banks can do so even if they do not underreport or understate their estimates of minimum capital requirements.

While Barth, Caprio, and Levine (2012, Ch. 7) and Kane (2012) recognize that some aspects of the Dodd-Frank Act might be beneficial, they conclude that overall it is an insufficient response to the crisis.7 Broadly speaking, our findings support their views, but our

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7 Caprio, Demirgüç-Kunt, and Kane (2010), Dewatripont, Rochet, and Tirole (2010, Ch. 2–4), Levine (2010b), Barth, Caprio,
argument differs from theirs. Indeed, we find that a specific section of the Dodd-Frank Act (i.e., the Volcker rule) is beneficial in partially mitigating the Basel framework’s drawback of allowing banks to take substantive tail risk in their trading portfolios without capital requirement penalties. In contrast, Barth, Caprio, and Levine argue that the Dodd-Frank Act is complex and vague, gives too much discretion to regulatory authorities in interpreting the intent of the law, and perpetuates the lack of accountability of regulators to the public. Accordingly, they suggest that the Dodd-Frank Act does not introduce reforms essential to reducing the likelihood and severity of future crises. Kane argues that the Dodd-Frank Act is inadequate in addressing incentive conflicts between, for example, regulatory officials and taxpayers. He suggests that these incentive conflicts will continue to allow financial institutions to extract subsidies from the safety net by aggressively engaging in innovative risk-taking activities.

Additionally, our findings support Admati and Pfleiderer (2012) who note that the Volcker rule is potentially effective in curbing socially-inefficient risk taking, but alone cannot make the financial system as safe as it should be. Moreover, they point out that there are systemic risk concerns if excessive leverage is permitted. Hence, they recommend a notable increase in the minimum capital requirements for banks. As an alternative to the Volcker rule, Duffie (2012) and Thakor (2012) also suggest such an increase.

We proceed as follows. Section 2 describes the model. Section 3 examines the effectiveness of the Basel framework and Volcker rule in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties. Section 4 assesses the robustness of our results. Section 5 suggests alternatives to the Basel framework and discusses the impact of the Volcker rule if any one of them is adopted. Section 6 concludes.

and Levine (2012, Ch. 8), and Kane (2012) offer recommendations on how to revise bank regulation in light of the crisis.
2. The model

Assume that uncertainty is described by a set of states \( \Omega = \{1, ..., S\} \). The probability of state \( s \) is \( p_s \equiv P[s] > 0 \). There are \( J \) risky assets \( (j = 1, ..., J) \) and a risk-free asset \( (j = J + 1) \) whose returns are given by a \((J + 1) \times S\) matrix \( R \). The return of asset \( j \) in state \( s \) is \( R_{js} \). The \((J + 1) \times 1\) vector of expected asset returns is denoted by \( \mu \) where the \( j \)th entry represents asset \( j \)'s expected return. The \( J \times J \) variance-covariance matrix of risky asset returns is denoted by \( \Sigma \) where the entry in row \( j_1 \) and column \( j_2 \) represents the covariance between the returns on assets \( j_1 \) and \( j_2 \). We assume that \( \text{rank}(\Sigma) = J \).

Let \( 1 \) denote the \((J + 1) \times 1\) vector \([1 \ \cdot \cdot \cdot \ 1]^T\). A portfolio is a \((J + 1) \times 1\) vector \( w = [w_1 \ \cdot \cdot \cdot \ w_{J+1}]^T \) with \( w^T 1 = 1 \), where \( w_j \) represents the weight of asset \( j \). Note that a positive (negative) asset weight represents a long (short) position in the asset.

2.1. VaR

In defining VaR, we follow Rockafellar and Uryasev (2002, Proposition 8). Fix a confidence level \( \alpha \in (1/2, 1) \). Let \( \tilde{R}_w \) denote the random return of portfolio \( w \). Let \( z_{1,w} < z_{2,w} < \cdots < z_{N_w,w} \) denote the ordered values that \( \tilde{z}_w \equiv -\tilde{R}_w \) can take across all states where \( N_w \leq S \) is the number of these values. Define \( n_\alpha \) as the unique index number with:

\[
\sum_{n=1}^{n_\alpha} p_{n,w} \geq \alpha > \sum_{n=1}^{n_\alpha - 1} p_{n,w},
\]

where \( p_{n,w} \equiv P[\tilde{z}_w = z_{n,w}] \). Note that while \( n_\alpha \) depends on \( w \), for brevity we write \( 'n_\alpha' \) instead of \( 'n_{\alpha,w}' \). Portfolio \( w \)'s VaR at the 100\(\alpha\)% confidence level is given by:

\[
V_{\alpha,w} \equiv z_{n_\alpha,w}.
\]

Eqs. (1) and (2) imply that:

\[
P[\tilde{R}_w \geq -V_{\alpha,w}] = P[\tilde{z}_w \leq z_{n_\alpha,w}] \geq \alpha,
\]

\[
P[\tilde{R}_w > -V_{\alpha,w}] = P[\tilde{z}_w < z_{n_\alpha,w}] < \alpha.
\]
Pérignon and Smith (2010) find that 73% of the banks who report their VaR estimation methodologies use historical simulation (see also Committee on the Global Financial System (2005)). Accordingly, we use historical simulation to estimate VaR.

2.2. CVaR

In defining CVaR, we again follow Rockafellar and Uryasev (2002, Proposition 8). Portfolio \( w \)'s CVaR at the 100\( \alpha \)% confidence level is given by:

\[
C_{\alpha,w} \equiv \frac{1}{1 - \alpha} \left[ \left( \sum_{n=1}^{n_{\alpha}} p_{n,w} - \alpha \right) z_{n_{\alpha},w} + \sum_{n=n_{\alpha}+1}^{N_w} p_{n,w} z_{n,w} \right].
\] (5)

Eqs. (2) and (5) imply that: (a) \( C_{\alpha,w} \geq V_{\alpha,w} \); and (b) \( C_{\alpha,w} > V_{\alpha,w} \) if \( P[\tilde{R}_w < -V_{\alpha,w}] > 0 \).

As in the case of VaR, we use historical simulation to estimate CVaR.

2.3. Stressed VaR

The definition of stressed VaR is similar to the definition of VaR, but uses a set of \( \mathcal{S} \) stressed states, denoted by \( \mathcal{O} \), where \( \mathcal{O} \subset \Omega \). By construction, at least some of the assets suffer sizeable losses in these states. The probability of state \( s \) conditional on \( \mathcal{O} \) is denoted by \( p_{s} \equiv P[s | \mathcal{O}] \). Fix a confidence level \( \pi \in (1/2, 1) \) and a portfolio \( w \). Let \( \tilde{z}_1 < \tilde{z}_2 < \cdots < \tilde{z}_{N_w} \) denote the ordered values that \( \tilde{z}_w \) can take in the stressed states where \( N_w \leq \mathcal{S} \) is the number of these values. Define \( \pi_{\pi,w} \) as the unique index number with:

\[
\sum_{n=1}^{\pi_{\pi,w}} p_{n,w} \geq \pi > \sum_{n=1}^{\pi_{\pi,w} - 1} p_{n,w}.
\] (6)

Note that while \( \pi_{\pi,w} \) depends on \( w \), for brevity we write \( \pi_{\pi} \) instead of \( \pi_{\pi,w} \). Portfolio \( w \)'s stressed VaR at the 100\( \pi \)% confidence level is given by:

\[
\bar{V}_{\pi,w} \equiv \tilde{z}_{\pi_{\pi,w}}.
\] (7)

Eqs. (6) and (7) imply that:

\[
P[\tilde{R}_w \geq -\bar{V}_{\pi,w} | \mathcal{O}] = P[\tilde{z}_w \leq \tilde{z}_{\pi_{\pi,w}} | \mathcal{O}] \geq \pi,
\] (8)

\[
P[\tilde{R}_w > -\bar{V}_{\pi,w} | \mathcal{O}] = P[\tilde{z}_w < \tilde{z}_{\pi_{\pi,w}} | \mathcal{O}] < \pi.
\] (9)
In estimating stressed VaR, the Basel Committee on Banking Supervision (2011, p. 15) suggests the use of a “12-month period relating to significant losses in 2007/2008.” Accordingly, we use the period January 2008–December 2008 to compute stressed VaR. Similar results (available upon request) are obtained when using the period of either January 2007–December 2007 or July 2007–June 2008.

2.4. Losses in stress testing events

While in practice there are many methods for stress testing a portfolio (see, e.g., Committee on the Global Financial System (2005)), we use scenario analysis to set risk exposure limits. In doing so, we base the scenarios that are analyzed on certain historical events. Therefore, we refer to them as ‘stress testing events.’

There are \( K \) stress testing events \( (k = 1, \ldots, K) \). Let \( R_k \) be the \((J+1) \times 1\) vector of asset returns in stress testing event \( k \). Portfolio \( w \)’s loss in stress testing event \( k \) is given by:

\[
L_{k,w} = -w^\top R_k.
\]

Using Eqs. (2) and (10), \( L_{k,w} \) can be smaller than, equal to, or larger than \( V_{\alpha,w} \). Similarly, using Eqs. (5) and (10), \( L_{k,w} \) can be smaller than, equal to, or larger than \( C_{\alpha,w} \). Also, using Eqs. (7) and (10), \( L_{k,w} \) can be smaller than, equal to, or larger than \( V_{\pi,w} \).

2.5. A risk management system based on the Basel framework

We consider a risk management system based on the following constraints:

\[
V_{\alpha,w} + V_{\pi,w} \leq U^{V+V}, \tag{11}
\]

\[
L_{k,w} \leq U^{L_k}, k = 1, \ldots, K, \tag{12}
\]

\(^8\) In practice, the scenarios that are analyzed can alternatively be based on hypothetical events such as the U.S. economic outlook and oil price scenarios (see Committee on the Global Financial System (2005, Table 3a)). However, there is an important reason for us to focus on historical events. While there is a relatively clear-cut way of capturing the manner in which these events are used in practice, there is no general way of doing so for hypothetical events. For example, while traded asset returns during a historical event are precisely observed, we do not know exactly how users of hypothetical events relate them to asset returns (for a discussion of the subjectivity of such events, see, e.g., Hull (2007, pp. 212–213)).
where $U^{V+V}$ and $U^{L_{k}}$ are upper bounds on, respectively, the sum of VaR and stressed VaR and the loss in stress testing event $k$. Constraint (11) is motivated by the fact that the Basel framework sets the minimum capital requirement associated with a bank’s trading portfolio to be the sum of two terms involving VaR and stressed VaR. Constraint (12) is motivated by the fact that the framework requires banks to have a stress testing program in place. Additionally, the Basel Committee on Banking Supervision (2011, p. 10) requires banks to use a risk measurement system in conjunction with risk exposure limits. Since the Basel framework requires the use of VaR, stressed VaR, and stress testing to measure risk, constraints (11) and (12) are of particular interest. Indeed, banks now utilize VaR, stressed VaR, and stress testing to set risk exposure limits; see Basel Committee on Banking Supervision (2005, p. 12), Committee on the Global Financial System (2005, pp. 1, 15), and KPMG (2010).

2.6. Measuring tail risk

Previous work suggests the use of a mean-CVaR model to control tail risk (see, e.g., Agarwal and Naik (2004) and Bertsimas et al. (2004)). Of particular interest is the question of whether the use of a risk management system based on constraints (11) and (12) leads to the selection of portfolios with small efficiency losses relative to the mean-CVaR frontier.

\footnote{The first term is the maximum between: (i) a factor between three and four times the average VaR over the last sixty business days; and (ii) the VaR in the previous business day. The second term is the maximum between: (a) a factor also between three and four times the average stressed VaR over the last sixty business days; and (b) the stressed VaR in the previous business day. The size of these two factors is set by regulators and depends on the performance of bank risk management systems; see Basel Committee on Banking Supervision (2011, pp. 15–16). Specifically, regulators assess the performance of such systems by examining the number of days where trading losses exceed VaR. If this number is excessive for a given bank, then the bank faces additional capital requirement penalties arising from an increase in the size of the factors. The interaction of such penalties and the performance of bank risk management systems is an issue of interest but beyond the scope of our paper. Santos, Nogales, Ruiz, and Dijk (2012) examine this issue by developing a portfolio selection model based on the Basel framework that constrains the number of days where trading losses exceed VaR in order to avoid these penalties. Lucas (2001) models these penalties within the original Basel framework for trading portfolios; see Basel Committee on Banking Supervision (1996a, 1996b). He finds that the penalties do not provide incentives for banks to truthfully report the VaRs of their trading portfolios (banks end up underreporting their VaRs as noted earlier). Our paper assumes that banks report VaRs and stressed VaRs that are sufficiently close to their ‘true’ VaRs and stressed VaRs. Assuming that banks underreport them would arguably strengthen our result that, regardless of the absence or presence of the Volcker rule, the Basel framework is ineffective in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties.}
Here, a portfolio belongs to the *mean-CVaR frontier* if there is no portfolio with the same expected return and a smaller CVaR. Also, a portfolio’s *efficiency loss* is the difference between: (1) its CVaR and (2) the CVaR of the portfolio on this frontier with the same expected return. Hence, a portfolio’s efficiency loss represents the increase in CVaR arising from selecting it instead of the portfolio with the same expected return that has minimum CVaR.

**2.7. Methodology**

Fig. 1 summarizes our methodology. Step 1 sets the constraints on asset weights. Specifically, we constrain the weight of each asset to be between lower bound $w_l = -50\%$ and upper bound $w_u = 150\%$.\(^{10}\) Hence, short selling is allowed.\(^{11}\) Letting a portfolio’s leverage ratio be defined as the sum of its positive weights, we also preclude the selection of portfolios with leverage ratios that exceed $400\%$.\(^{12}\) We should emphasize that the possibility of short selling and the range of leverage ratios allowed in our paper are realistic in the context of the trading portfolios of large U.S. banks. Consider the seventeen U.S. depository institutions with total assets of $100$ billion or more and positive trading assets as of December 31, 2011 (see Federal Deposit Insurance Corporation (2012)). First, the trading portfolios of sixteen out of these seventeen institutions involve short selling. Second, of the sixteen institutions for which leverage ratios are well-defined, eleven have leverage ratios less than $400\%$.\(^{13}\) Third, the average leverage ratio across these sixteen institutions is $462\%$.

Step 2 chooses the confidence level and bounds for constraints (11) and (12). Based on

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\(^{10}\)Similar results (available upon request) are obtained when $w_l = -100\%$ and $w_u = 200\%$.

\(^{11}\)The results when short selling is disallowed are available upon request. They differ from those when it is allowed in that the Basel framework is more effective in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties. However, consistent with our main findings: (1) the Basel framework is still not fully effective in preventing banks from doing so; and (2) the Volcker rule still only partially mitigates the ineffectiveness of the Basel framework.

\(^{12}\)Similar results (available upon request) are obtained when we preclude the selection of portfolios with leverage ratios that exceed $500\%$.

\(^{13}\)The difference between trading assets and trading liabilities is negative for one institution. Hence, its leverage ratio is not well-defined.
the requirements of the Basel framework, we use a confidence level of 99% to compute VaR and stressed VaR (i.e., \( \alpha = \bar{\sigma} = 99\% \)). For consistency, we also use a confidence level of 99% to compute CVaR. In regard to the bounds, we consider two cases. The first case involves bounds that do not depend on the required expected return, which we refer to as ‘fixed bounds.’ The second case involves bounds that depend on the required expected return, which we refer to as ‘variable bounds.’ In this case, we assume that:

\[ U_{E}^{V+\nabla} = V_{\alpha, w_{\alpha, E}} + \nabla_{\pi, w_{\alpha, E}} \]
\[ U_{E}^{L_{k}} = L_{k, w_{\alpha, E}}, \quad k = 1, \ldots, K, \]

where \( w_{\alpha, E} \) denotes the portfolio on the mean-CVaR frontier with a confidence level of \( \alpha \) and an expected return of \( E \).\(^{14}\)

While the minimum required expected return \( E \) is assumed to be the risk-free rate, the maximum required expected return \( \bar{E} \) is found in Step 3. Specifically, \( \bar{E} \) is assumed to be the maximum feasible expected return in the presence of constraints on the: (a) asset weights; (b) sum of VaR and stressed VaR; and (c) losses in stress testing events. Note that the value of the bounds as well as the set of available assets generally affect the size of \( \bar{E} \). For example, the value of \( E \) in presence of the Volcker rule is noticeably smaller than that in its absence.

Step 4 uses the values of \( E \) and \( \bar{E} \) to calculate \( \delta \equiv (\bar{E} - E)/100 \). Using the value of \( \delta \), Step 5 creates a grid of 101 expected returns \( \{E_{i}\}_{i=0}^{100} \) where \( E_{i} \equiv E + \delta i \) for \( i = 0, 1, \ldots, 100 \). Note that this grid ranges from \( E_{0} = E \) to \( E_{100} = \bar{E} \). Since the set of available assets generally affects the size of \( E \), it also generally affects the values in the grid of expected returns. In particular, the grid of expected returns in the presence of the Volcker rule noticeably differs

\(^{14}\)Note that the use of the bounds given by Eqs. (13) and (14): (i) allows (but typically do not force) the selection of portfolio \( w_{\alpha, E} \), which has by construction an expected return of \( E \) and a zero efficiency loss; and (ii) precludes (as much as possible) portfolios with an expected return of \( E \) that have the greatest efficiency losses, thereby leading to the smallest maximum efficiency loss. Generally, when bounds other than \( U_{E}^{V+\nabla} \) and \( \{L_{k}^{E}\}_{k=1}^{K} \) are used, either (i) or (ii) do not hold. First, the use of smaller values for the bounds precludes the selection of portfolio \( w_{\alpha, E} \). Second, the use of larger values for the bounds typically leads to a larger maximum efficiency loss. The use of the bounds given by Eqs. (13) and (14) is thus appealing.
from than that in its absence.

In Step 6, for each level of expected return $E_i$, we find the maximum efficiency loss $M_i$. Fig. 2 illustrates how $M_i$ is determined in the case where the Volcker rule is absent (the case where the Volcker rule is present is similar). The curve represents portfolios on the mean-CVaR frontier. Point $p_i^{\text{max}}$ represents the portfolio that has an expected return of $E_i$, satisfies the constraints on asset weights, the sum of VaR and stressed VaR, and losses in stress testing events, and has maximum CVaR, denoted by $C_i^{\text{max}}$. Point $p_i^{\text{min}}$ represents the portfolio that has the same expected return, satisfies the constraints and asset weights, and has minimum CVaR (i.e., the portfolio on the mean-CVaR frontier with an expected return of $E_i$), denoted by $C_i^{\text{min}}$. Since $M_i = C_i^{\text{max}} - C_i^{\text{min}}$, $M_i$ is the maximum increase in CVaR allowed by the constraints on the sum of VaR and stressed VaR as well as on the losses in stress testing events given a required expected return of $E_i$. By construction, we have $M_i \geq 0$. If $M_i = 0$, then the constraints on the sum of VaR and stressed VaR as well as on the losses in stress testing events are fully effective in controlling tail risk when the required expected return is $E_i$. In contrast, if $M_i > 0$, then the constraints are not fully effective in doing so. Next, the average of $\{M_i\}_{i=0}^{100}$, referred to as the average efficiency loss, is determined. The largest value of $\{M_i\}_{i=0}^{100}$, referred to as the largest efficiency loss, is also determined.

In Step 7, for each level of expected return $E_i$ with $C_{\alpha,w_{\alpha,E_i}} > 1\%$, we calculate the relative efficiency loss $REL_i \equiv M_i/C_{\alpha,w_{\alpha,E_i}}$.\footnote{There are two difficulties in determining relative efficiency losses. First, portfolios on the mean-CVaR frontier with expected returns close to the risk-free return have negative CVaRs and thus their relative efficiency losses are negative. Second, even when the CVaRs of certain portfolios on the mean-CVaR frontier are positive, they could be arbitrarily close to zero, resulting in arbitrarily large relative efficiency losses. While these two difficulties do not affect the size of, for example, the median relative efficiency loss, they do affect the size of the average and largest relative efficiency losses. Accordingly, in order to circumvent such difficulties, we determine $REL_i$ only for levels of expected return $E_i$ with $C_{\alpha,w_{\alpha,E_i}} > 1\%$.} Note that $REL_i$ is the ratio between efficiency loss $M_i$ and the CVaR of the minimum CVaR portfolio with an expected return of $E_i$. Let $I_{REL} \equiv \{i \in \{0, ..., 100\} : C_{\alpha,w_{\alpha,E_i}} > 1\%\}$. Since we compute $REL_i$ only for any $i \in I_{REL}$...
and $M_i \geq 0$, we have $REL_i \geq 0$. We also determine average and largest relative efficiency losses from $\{REL_i\}_{i \in I_{REL}}$.

In Step 8, for each level of expected return $E_i$ in the grid for the case where the Volcker rule is present, we compute the reduction in maximum efficiency loss $RM_i$ arising from the presence of this rule. Fig. 3 illustrates how $RM_i$ is determined. The solid curve represents portfolios on the mean-CVaR frontier when the Volcker rule is absent, whereas the dashed curve represents portfolios on the mean-CVaR frontier when the Volcker rule is present. Consider the case where the Volcker rule is absent. Point $p_{i}^{\text{max,ab}}$ represents the portfolio that has an expected return of $E_i$, satisfies constraints on the asset weights, sum of VaR and stressed VaR, and losses in stress testing events, and has maximum CVaR, denoted by $C_{i}^{\text{max,ab}}$. Point $p_{i}^{\text{min,ab}}$ represents the portfolio that has the same expected return, satisfies the asset weight constraints, and has minimum CVaR, denoted by $C_{i}^{\text{min,ab}}$. The maximum efficiency loss is $M_{i}^{\text{ab}} \equiv C_{i}^{\text{max,ab}} - C_{i}^{\text{min,ab}}$. Consider now the case where the Volcker rule is present. Points $p_{i}^{\text{min,pr}}$ and $p_{i}^{\text{max,pr}}$ represent, respectively, the portfolios with minimum and maximum CVaRs. The CVaRs of such portfolios are denoted by $C_{i}^{\text{min,pr}}$ and $C_{i}^{\text{max,pr}}$. The maximum efficiency loss is $M_{i}^{\text{pr}} \equiv C_{i}^{\text{max,pr}} - C_{i}^{\text{min,pr}}$. Hence, the reduction in maximum efficiency loss is $RM_i \equiv M_{i}^{\text{ab}} - M_{i}^{\text{pr}}$.\footnote{Note that some of the notation in Fig. 3 expands upon the notation in Fig. 2. For example, depending on whether the Volcker rule is absent or present, we use, respectively, $C_{i}^{\text{max,ab}}$ and $C_{i}^{\text{max,pr}}$ in Fig. 3, whereas we use $C_{i}^{\text{max}}$ in Fig. 2.} If $RM_i < 0$, then the presence of the Volcker rule reduces the effectiveness of the constraints on the sum of VaR and stressed VaR as well as on the losses in stress testing events in controlling tail risk when the required expected return is $E_i$. If $RM_i = 0$, then the presence of the Volcker rule does not affect the effectiveness of such constraints when the required expected return is $E_i$. If $RM_i > 0$, then the presence of the Volcker rule mitigates the ineffectiveness of the constraints when the required expected return is $E_i$.\footnote{In the case of fixed bounds, observe that $RM_i \geq 0$. This result can be understood as follows. Since the minimum CVaR is non-increasing in the number of available assets, we obtain $C_{i}^{\text{min,ab}} \leq C_{i}^{\text{min,pr}}$. Multiplying this inequality by $-1$, we have} Next, the average of $\{RM_i\}_{i=0}^{100}$, referred to as the average reduction in efficiency
loss, is determined. The largest value of \( \{RM_i\}_{i=0}^{100} \), referred to as the largest reduction in efficiency loss, is also determined.

In Step 9, for each level of expected return \( E_i \) in the grid for the case where the Volcker rule is present, we compute the relative reduction in maximum efficiency loss \( RRM_i \) arising from the presence of this rule. In doing so, we let \( RRM_i \equiv RM_i/M_{i}^{ab} \) for any \( i \in I_{RRM} \), where \( I_{RRM} \equiv \{i \in \{0, ..., 100\} : M_i^{ab} > 0\} \) (note that \( RRM_i \) is undefined when \( M_i^{ab} = 0 \)). Using the definitions of \( RRM_i \) and \( RM_i \), we obtain \( RRM_i = (M_i^{ab} - M_i^{pr})/M_i^{ab} = 1 - M_i^{pr}/M_i^{ab} \).

Since \( M_i^{pr} \geq 0 \) and \( RRM_i \) is determined only for any \( i \in I_{RRM} \), we have \( RRM_i \leq 1 \). If \( RRM_i < 0 \), then the presence of the Volcker rule reduces the effectiveness of the constraints on the sum of VaR and stressed VaR as well as on the losses in stress testing events in controlling tail risk when the required expected return is \( E_i \). If \( RRM_i = 0 \), then the presence of the Volcker rule does not affect the effectiveness of such constraints when the required expected return is \( E_i \). If \( 0 < RRM_i < 1 \), then the presence of the Volcker rule partially mitigates the ineffectiveness of the constraints when the required expected return is \( E_i \). If \( RRM_i = 1 \), then the presence of the Volcker rule fully mitigates the ineffectiveness of the constraints when the required expected return is \( E_i \). Next, the average of \( \{RRM_i\}_{i \in I_{RRM}} \), referred to as the average relative reduction in efficiency loss, is determined. The largest value of \( \{RRM_i\}_{i \in I_{RRM}} \), referred to as the largest relative reduction in efficiency loss, is also determined. Subsequent analysis is based on the values of \( \{M_i\}_{i=0}^{100} \), \( \{REL_i\}_{i \in I_{REL}} \), \( \{RM_i\}_{i=0}^{100} \), and \( \{RRM_i\}_{i \in I_{RRM}} \).

An examination of maximum efficiency losses captures the idea of being agnostic regard-

\[-C_i^{\min,ab} \geq -C_i^{\min,pr}.\]

Additionally, since the maximum CVaR is non-decreasing in the number of available assets, we obtain \( C_i^{\max,ab} \geq C_i^{\max,pr} \). Adding the last two inequalities and using the definitions of \( M_i^{ab} \) and \( M_i^{pr} \), we have \( C_i^{\max,ab} - C_i^{\min,ab} \geq M_i^{ab} \geq M_i^{pr} = C_i^{\max,pr} - C_i^{\min,pr} \). It follows that \( RM_i = M_i^{ab} - M_i^{pr} \geq 0 \). However, in the case of variable bounds, we possibly have \( RM_i < 0 \). This result can be understood by noting that the variable bounds in the absence of the Volcker rule possibly differ from those in its presence. Hence, we might have \( C_i^{\max,ab} < C_i^{\max,pr} \). It follows that we might have \( M_i^{ab} < M_i^{pr} \) and thus \( RM_i = M_i^{ab} - M_i^{pr} < 0 \).

\(^{18}\)As a robustness check, we use the interquartile range of expected returns \( \{E_i\}_{i=25}^{75} \) instead of the entire range in determining such values. The results (available upon request) are similar to those reported for the entire range.
ing the portfolio selection model that is used in the presence of constraints (11) and (12).\textsuperscript{19} The motivation for this idea is twofold. First, we are interested in exploring the effectiveness of these constraints in preventing the selection of portfolios with substantive efficiency losses without making any assumption on the portfolio selection model that is used in their presence. If the constraints preclude the selection of all such portfolios, then they are effective in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties regardless of the model that banks use to select them. However, if the constraints allow the selection of such portfolios, then they are ineffective in preventing banks from doing so. Second, while the use of risk management constraints by banks is apparent, we do not know the exact models that they utilize for portfolio selection. The size of trading losses in the crisis suggests that trading portfolio managers have incentives to take substantive tail risk. Indeed, subject to existing constraints, these managers might have incentives to take on as much tail risk as possible in attempting to realize relatively high returns (see Lucas (2001) and Hull (2007, p. 198)). For example, generous deposit insurance and compensation schemes might lead such managers to take excessive risks (see Cai, Cherny, and Milbourn (2010), Barth, Caprio, and Levine (2012, Ch. 3), and Kane (2012)).\textsuperscript{20}

2.8. Optimization inputs

In assessing the Basel framework, we consider a problem of wealth allocation among the following assets: (a) Treasury bills (assumed to be risk-free); (b) Treasury bonds with five maturity ranges (1–3, 3–5, 5–10, 10–15, and 15+ years); (c) Agency bonds with the same maturity ranges; and (d) the six size/book-to-market-based Fama-French equity portfolios.

\textsuperscript{19}Note that we are not agnostic regarding the model that should be used to control tail risk in the absence of such constraints. The mean-CVaR model is used to find efficiency losses.

\textsuperscript{20}Volcker (2012, p. 132) suggests that the restrictions on the proprietary trading activities of U.S. banks are “an important step to deal with risk, conflicts of interest and, potentially, compensation practices as well.”
Similarly, in assessing the impact of the Volcker rule, we consider a problem of wealth allocation among these assets. More precisely, we assess this impact by examining the effect of removing the Fama-French equity portfolios from consideration on the size of the efficiency losses.\textsuperscript{21} In Section 4, we extend our analysis by using six (instead of five) maturity ranges for the Treasury and Agency bonds and the ten size-based (instead of the six size/book-to-market-based) Fama-French equity portfolios.\textsuperscript{22}

In our base case, we consider four stress testing events: (i) U.S. stock market crash of 1987 (October 14–19); (ii) U.S. interest rate hike of 1994 (January 31–December 13); (iii) terrorist attacks of 2001 (September 11–17); and (iv) subprime crisis of 2007–09 (October 1, 2007–February 27, 2009). For robustness checks, we consider four additional stress testing events: (v) Russian crisis of 1998 (August 17–September 21); (vi) LTCM collapse of 1998 (October 7); (vii) dot-com slowdown of 2001–02 (March 10, 2001–October 9, 2002); and (viii) Greek crisis of 2010 (April 1–May 31).\textsuperscript{23}

The distribution of asset returns is based on daily data during 1987–2011.\textsuperscript{24} Returns on Treasury bills and Fama-French equity portfolios are obtained from Kenneth French’s website. Returns on Treasury and Agency bonds are extracted from Bloomberg by using the Bank of America Merrill Lynch Treasury and Agency bond indices.\textsuperscript{25} The first five rows of Table 1 present summary statistics on asset returns. These statistics are scaled to a period of ten trading days. In doing so, we multiply the average daily return by ten and

\textsuperscript{21}The Volcker rule explicitly permits banks to engage in trading activities that involve Treasury bills, Treasury bonds, and Agency bonds; see Dodd-Frank Act (2010, p. 668). While the Volcker rule establishes exceptions for trading other assets (involving, for example, hedging and market-making activities), an examination of the effect of such exceptions is beyond the scope of our paper. However, to the extent that these exceptions weaken the impact of the Volcker rule, our findings still suggest that the Volcker rule does not fully address the ineffectiveness of the Basel framework in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties.

\textsuperscript{22}We also confirm our findings in the case where Treasury bills are removed from consideration (available upon request).

\textsuperscript{23}The time periods for the stress testing events follow BarraOne (a leading risk management platform); see: <www.msci.com/products/risk_management_analytics/barraone/barraone_historical_stress_testing_scenarios.html>.

\textsuperscript{24}Similar results (available upon request) are obtained when using the period 1997–2011.

\textsuperscript{25}An examination of the extent to which our results are affected by the presence of estimation risk and non-stationarity is beyond the scope of our paper. For work that recognizes estimation risk in VaR and CVaR, see, e.g., the November 2000 issue of the Journal of Empirical Finance, the July 2002 issue of the Journal of Banking and Finance, and Pritsker (2006).
the remaining statistics by the square root of ten.\textsuperscript{26} With the exception of small-growth stocks, assets involving stocks have larger average returns than those involving bonds.\textsuperscript{27} Also, assets involving stocks have larger standard deviations, VaRs, CVaRs, and stressed VaRs than assets involving bonds.\textsuperscript{28} Moreover, the CVaR of each risky asset is larger than its VaR. Lastly, the stressed VaR of each risky asset is larger than its CVaR.

The last four rows show the returns in stress testing events. Treasury bills have positive returns in such events. Treasury bonds with a maturity range of 1–3 years have a negative return in one event (U.S. interest rate hike of 1994), and positive returns in the remaining three events. Treasury bonds with the other maturity ranges have negative returns in two events (U.S. stock market crash of 1987 and U.S. interest rate hike of 1994), and positive returns in the remaining two events. Agency bonds with all maturity ranges also have negative returns in two events (again, U.S. stock market crash of 1987 and U.S. interest rate hike of 1994), and positive returns in the remaining two events. The Fama-French equity portfolios have negative returns in all the four events.

3. Results

This section presents our results.

3.1. Assessment of the Basel framework

We begin by assessing the effectiveness of the Basel framework in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties. As noted earlier, we do so by considering a risk management system based on constraints

\textsuperscript{26}The Basel framework explicitly allows banks to estimate VaR for a period of ten trading days by multiplying an estimate of VaR for a period of one trading day by the square root of ten; see Basel Committee on Banking Supervision (2011, p. 14).

\textsuperscript{27}Since the average return on small/growth stocks is smaller than those of some assets involving bonds (e.g., Treasury bonds with 15+ years), we consider an additional case where the expected return on small/growth stocks is adjusted upwards. In doing so, we add one basis point to the sample daily returns on small/growth stocks. When scaled to a period of ten trading days, the expected return on small/growth stocks becomes 0.40%, which exceeds those of the assets involving bonds. The results in such a case (available upon request) are similar to those presented when using the unadjusted sample average return on small/growth stocks.

\textsuperscript{28}The standard deviation of the return on Treasury bills is reported as zero since they are assumed to be risk-free.
that limit the sum of VaR and stressed VaR as well as losses in stress testing events; see Eqs. (11) and (12). Also, we assume that the seventeen assets of Table 1 are available.

Table 2 provides the results with such assets. The first four columns consider fixed-bound constraints. The reported fixed bounds on the sum of VaR and stressed VaR shown below $U^{V+V}$ are scaled to a period of ten trading days by multiplying the daily bounds by the square root of ten. In contrast, the reported fixed bounds on the losses in stress testing events shown below $U^{L_k}$ apply to the periods of time capturing the events as defined in Table 1.\textsuperscript{29}

The first two rows report average and largest efficiency losses that are scaled to a period of ten trading days by multiplying the daily efficiency losses by the square root of ten. Note that losses are sizeable for all values of the bounds. Furthermore, losses are smaller when the lower values of the bounds are used. The next two rows indicate that average and largest relative efficiency losses are also sizeable and either similar or smaller when the lower values of the bounds are used.\textsuperscript{30}

We examine the distance between any two given portfolios $w^1$ and $w^2$ by computing

$$\frac{|w^1 - w^2|}{\sqrt{J+1}} = \left[\sum_{j=1}^{J+1} (w^1_j - w^2_j)^2 / (J+1)\right]^{1/2};$$

here, in the numerator of the left-hand side of this equation, ‘$|$’ denotes the Euclidean norm. Let $D_i$ denote the distance between the portfolio with maximum efficiency loss and the portfolio on the mean-CVaR frontier when the required expected return is $E_i$. The average and largest values of $\{D_i\}_{i=0}^{100}$ are referred to as, respectively, \textit{average distance} and \textit{largest distance}. The fifth and sixth rows report that

\textsuperscript{29}While the fixed bound values in the table are utilized for illustrative purposes, similar results have been obtained when other reasonable values are used.

\textsuperscript{30}Note that the largest relative efficiency loss with $U^{V+V} = U^{L_k} = 4\%$ is slightly larger than that with $U^{V+V} = 4\%$ and $U^{L_k} = 20\%$. While this result might seem counterintuitive (a lower value of $U^{L_k}$ corresponds to a tighter constraint on the loss in stress testing event $k$), it can be understood by noting three facts. First, the maximum feasible expected return with $U^{V+V} = U^{L_k} = 4\%$ (i.e., 0.4138\%) is slightly smaller than that with $U^{V+V} = 4\%$ and $U^{L_k} = 20\%$ (i.e., 0.4141\%). Second, it follows that the grid of required expected returns with $U^{V+V} = U^{L_k} = 4\%$ differs slightly from that with $U^{V+V} = 4\%$ and $U^{L_k} = 20\%$ (see Steps 4 and 5 of our methodology in Section 2.7). Third, the constraints on the losses in stress testing events end up not binding at the levels of expected return where the relative efficiency losses are largest. Due to these three facts, the largest relative efficiency loss turns out to be slightly larger when $U^{V+V} = U^{L_k} = 4\%$.  

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these distances are also sizeable and smaller when the lower values of the bounds are used.

The last row reports the maximum feasible expected return. Note that this expected return is scaled to a period of ten trading days by multiplying the daily maximum feasible expected return by ten. Not surprisingly, the maximum feasible expected return is smaller when the lower values of the bounds are used.\(^\text{31}\)

The last column considers variable-bound constraints. Three results are worth noting. First, the losses and distances with variable-bound constraints are considerably smaller than those with fixed-bound constraints.\(^\text{32}\) Second, the losses are still quite sizeable.\(^\text{33}\) Third, the variable-bound constraints allow the selection of portfolios with larger expected returns than the ones allowed by fixed-bound constraints. Hence, the use of variable bounds is beneficial relative to the use of fixed bounds.

Column (1) of Fig. 4 shows a box plot of maximum efficiency losses with variable-bound constraints. The three horizontal lines in the box represent the lower quartile, median, and upper quartile of losses. The dashed vertical lines extending from each end of the box show the range of losses. Hence, the horizontal line at the bottom (top) of the lower (upper) dashed vertical line represents the lowest (highest) value of the loss. Note that the median loss is about 1.5\%. Furthermore, the range of losses is quite sizeable.

Next, we illustrate the tail risk of: (i) portfolios on the mean-CVaR frontier; and (ii) portfolios with maximum efficiency losses. For brevity, consider required expected returns of \(E_{33}\) and \(E_{67}\), which are, respectively, 33\% and 67\% of the way between the risk-free rate (\(E_0\))

\(^{31}\)Since Table 2 rounds the reported values by using two decimal places, the maximum feasible expected return is reported as 0.41\% in the first two columns. However, the maximum feasible expected return with \(U^V + V = U^L = 4\% (0.4138\%)\) is indeed slightly smaller than that with \(U^V + V = 4\%\) and \(U^L = 20\% (0.4141\%); see footnote 30.

\(^{32}\)Intuitively, while fixed-bound constraints are tight for only the largest feasible levels of expected return, variable-bound constraints are tight for all feasible levels. Hence, the losses and distances are smaller with variable-bound constraints.

\(^{33}\)Since the reported efficiency losses are scaled to a period of ten trading days, the losses are indeed still quite sizeable. For example, the ten-day average efficiency loss of 1.19\% corresponds to a one-year average efficiency loss of 5.95\% (= 1.19\% \times \sqrt{250/\sqrt{10}}). Here, we assume that one year corresponds to 250 trading days. Like efficiency losses, relative efficiency losses are still quite sizeable. For example, the average relative efficiency loss is about 31\%, indicating that, on average, the use of the constraints allows an increase in CVaR of 31\% (relative to the CVaRs of the minimum CVaR portfolios).
and the maximum feasible expected return ($E$). The first two rows of Table 3 show return and risk statistics for: (i) the portfolio on the mean-CVaR frontier with an expected return of $E_{33}$; and (ii) the portfolio with this expected return that has the maximum efficiency loss when variable-bound constraints are used. Note that the latter portfolio has an efficiency loss of 2.49% [$= 6.71\% - 4.22\%$] and a relative efficiency loss of 59.05% [$= 2.49\%/4.22\%$] even though it has: (a) the same sum of VaR and stressed VaR as the portfolio on the mean-CVaR frontier; and (b) losses in stress testing events that do not exceed those of the portfolio on the mean-CVaR frontier. It follows that the two aforementioned portfolios differ in terms of tail risk (compare their CVaRs of 6.71% and 4.22%) but have the same minimum capital requirements within the Basel framework since the sum of VaR and stressed VaR is 11.42% for both portfolios. Hence, this framework allows banks to take substantive tail risk in their trading portfolios without capital requirement penalties. The last two rows of Table 3 show similar results in the case where an expected return of $E_{67}$ is used. In this case, the efficiency loss is 1.67% [$= 15.82\% - 14.15\%$], whereas the relative efficiency loss is 11.77% [$= 1.67\%/14.15\%$].

Of particular interest are the CVaRs of portfolios: (a) on the mean-CVaR frontier; and (b) with maximum efficiency losses. For brevity, we focus on the case where variable-bound constraints are imposed. The CVaRs of portfolios on the mean-CVaR frontier range from $-0.05\%$ to $63.73\%$ with the median CVaR being $8.49\%$. The CVaRs of portfolios with maximum efficiency losses also range from $-0.05\%$ to $63.73\%$, but the median CVaR is $10.39\%$. Hence, the median CVaR of portfolios with maximum efficiency losses exceeds

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34 Notice that a negative loss in a stress testing event corresponds to a positive return; see Eq. (10).

35 Note that the minimum CVaR of portfolios with maximum efficiency losses (i.e., $-0.05\%$) coincides with the minimum CVaR of portfolios on the mean-CVaR frontier (the minimum CVaR portfolio is the risk-free asset). This result is driven by the fact that the portfolio with maximum efficiency loss when the required expected return is $E$ (i.e., the minimum required expected return) has a zero efficiency loss. The maximum CVaR of portfolios with maximum efficiency losses (i.e., $63.73\%$) also coincides with the maximum CVaR of portfolios on the mean-CVaR frontier. This result is driven by two facts. First, the portfolio with maximum efficiency loss when the required expected return is $E$ (i.e., the maximum required expected return) has a zero efficiency loss. Second, the portfolios with maximum efficiency losses when the required expected returns differ from $E$ have
the median CVaR of portfolios on the mean-CVaR frontier by 1.90% \(= 10.39\% - 8.49\%\) in absolute terms, and 22.42% \(= 1.90\%/8.49\%\) in relative terms.

In sum, we find that the use of constraints (11) and (12) allow the selection of portfolios with substantive efficiency losses. This finding suggests that the Basel framework is ineffective in preventing banks from taking substantive risk in their trading portfolios without capital requirement penalties.\(^{36}\)

### 3.2. Impact of the Volcker rule

Next, we assess the extent to which the Volcker rule mitigates the ineffectiveness of the Basel framework. As noted earlier, we do so by comparing the extent to which constraints (11) and (12) allow the selection of portfolios with substantive efficiency losses when two different sets of available assets are used. The first set involves Treasury bills, Treasury bonds, Agency bonds, and the Fama-French equity portfolios (i.e., the seventeen assets in Table 1), whereas the second set involves only Treasury bills, Treasury bonds, and Agency bonds (i.e., eleven assets).

Table 4 provides the results with eleven assets. The first four columns consider fixed-bound constraints. The first four rows show that the losses are sizeable for all values of the bounds and typically smaller when the lower values of the bounds are used.\(^{37}\) However, the losses with eleven assets are notably smaller than the losses with seventeen assets (compare smaller CVaRs than the portfolio with maximum efficiency loss when the required expected return is \(\bar{E}\).

\(^{36}\)Note that we assess the Basel framework solely along the dimension of whether it is effective in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties. While there exist other important dimensions along which the Basel framework can be assessed, they are beyond the scope of our paper. For example, of interest is the extent to which it mitigates the pro-cyclicality (e.g., higher minimum capital requirements in recessions) of the original Basel framework for trading portfolios; see Basel Committee on Banking Supervision (1996a). Also of interest is the extent to which the use of a relatively short window for estimation purposes results in large errors in estimates of VaR and stressed VaR that, in turn, can lead to inadequate minimum capital requirements. For example, the use of a twelve-month period involving the crisis might not precisely capture the size of trading losses in future crises.

\(^{37}\)Note that the average relative efficiency loss with \(U^{V+\bar{V}} = 20\%\) and \(U^{L_k} = 4\%\) is larger than that with \(U^{V+\bar{V}} = U^{L_k} = 20\%\). While this result might seem counterintuitive (as noted earlier, the lower value of \(U^{L_k}\) corresponds to a tighter constraint on the loss in stress testing event \(k\)), it can be understood as follows. Observe that the maximum feasible expected return with \(U^{V+\bar{V}} = 20\%\) and \(U^{L_k} = 4\%\) is smaller than that with \(U^{V+\bar{V}} = U^{L_k} = 20\%\) (see the last row of Table 4). Since relative efficiency losses end up being larger for relatively small levels of expected return, the average relative efficiency loss is larger when \(U^{V+\bar{V}} = 20\%\) and \(U^{L_k} = 4\%\).
the first four columns of Tables 2 and 4). In contrast, the next two rows show that the
distances with eleven assets are larger than those with seventeen assets. Not surprisingly,
the last row indicates that the maximum feasible expected return with eleven assets is notably
smaller than that with seventeen assets.

The last column considers variable-bound constraints. Four main results can be seen.
First, the losses and distances with variable-bound constraints are considerably smaller than
those with fixed-bound constraints. Second, the losses are still not close to zero. Third, the
variable-bound constraints allow the selection of portfolios with larger expected returns than
the ones allowed by the fixed-bound constraints. Fourth, the losses with eleven assets are
notably smaller than the losses with seventeen assets (compare the last column of Tables 2
and 4).

Column (1) of Fig. 4 shows a box plot of maximum efficiency losses with variable-bound
constraints. Note that the median and upper quartile of losses with eleven assets are smaller
than those with seventeen assets (compare column (1) of Figs. 4 and 3, noting that the
y-axes of the two figures use different scales). However, the former quartiles are still not
close to zero.

In assessing the statistical significance of the difference between the distributions of losses
with seventeen and eleven assets (see column (1) of Figs. 4 and 3), we utilize: (i) the two-
sample Kolmogorov-Smirnov test and (ii) the Wilcoxon rank sum test. Using (i), we test
the null hypothesis that the cumulative distribution function (cdf) of losses when seventeen
assets are available coincides with the cdf of losses when eleven assets are available. The
alternative hypothesis is that the two cdfs differ. Similarly, using (ii), we test the null
hypothesis that the median of the distribution of losses when seventeen assets are available
equals the median when eleven assets are available. The alternative hypothesis is that the
two medians differ. In results available upon request, we find that both null hypotheses are
rejected at the 1% level. Hence, there is statistical significance to the result that losses with eleven assets are smaller than losses with seventeen assets.

The first two rows of Table 5 show return and risk statistics for: (i) the portfolio on the mean-CVaR frontier with an expected return of \( E_{33} \); and (ii) the portfolio with this expected return that has the maximum efficiency loss when variable-bound constraints are used. Note that the latter portfolio has an efficiency loss of 0.74% \([= 4.19\% - 3.45\%]\) and a relative efficiency loss of 21.40% \([= 0.74\%/3.45\%]\) even though it has: (a) the same sum of VaR and stressed VaR as the portfolio on the mean-CVaR frontier; and (b) losses in stress testing events that do not exceed those of the portfolio on the mean-CVaR frontier. It follows that the two aforementioned portfolios differ in terms of tail risk (compare their CVaRs of 4.19% and 3.35%) but have the same minimum capital requirements within the Basel framework since the sum of VaR and stressed VaR is 7.44% for both portfolios. Therefore, even in the presence of the Volcker rule, this framework allows banks to take substantive tail risk in their trading portfolios without capital requirement penalties. However, the efficiency and relative efficiency losses with eleven assets (i.e., 0.74% and 21.40%) are smaller than those with seventeen assets (i.e., 2.49% and 59.05%). Hence, the Volcker rule partially mitigates the ineffectiveness of the Basel framework in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties. The last two rows of Table 3 show similar results in the case where an expected return of \( E_{67} \) is used. In this case, the efficiency loss is 0.83% \([= 10.72\% - 9.89\%]\), whereas the relative efficiency loss is 8.37% \([= 0.83\%/9.89\%]\). Again, the efficiency and relative efficiency losses with eleven assets (i.e., 0.83% and 8.37%) are smaller than those with seventeen assets (i.e., 1.67% and 11.77%).\(^{38}\)

Table 6 examines the reductions in efficiency losses arising from the presence of the

\(^{38}\)Note that the value of \( E_{67} \) in the presence of the Volcker rule (i.e., 0.54%) equals the value of \( E_{33} \) in the absence of the Volcker rule; see the next to last row of Table 5 and the first row of Table 3. The equality of these two values is a coincidence (i.e., the two values possibly differ if other optimization inputs are used).
Volcker rule. The first four columns consider fixed-bound constraints. The first two rows indicate that the average and largest reductions in efficiency losses are sizeable for all values of the bounds. Moreover, such reductions are larger with the higher values of the bounds. The next row shows that the average relative reduction in efficiency loss ranges from 44.72% to 56.54%. The last row indicates that the largest relative reduction in efficiency loss is either very close or equal to 100%. The last column considers variable-bound constraints. The first two rows indicate that the average and largest reductions in efficiency losses are smaller than those with fixed-bound constraints. However, as the last two rows show, the average and largest relative reductions in efficiency losses at, respectively, 49.71% and 100%, are roughly comparable to those with fixed-bound constraints. Column (1) of Fig. 6 shows a box plot of relative reductions in efficiency losses with variable-bound constraints. Note that the median relative reduction is about 55%. Hence, while the Volcker rule notably reduces efficiency losses, it does not fully eliminate them.

While the Volcker rule has the benefit of partially mitigating the ineffectiveness of the Basel framework, it also has costs. As noted earlier, the maximum feasible expected return is notably smaller when the Volcker rule is present. More generally, the return-risk trade-off for trading portfolios is less favorable in the case where the Volcker rule is present relative to the case where it is absent. When the mean-CVaR frontier is used to describe such a trade-off, the effect of the Volcker rule can be seen in Tables 3 and 5. The first row of Table 3 indicates that in the absence of the Volcker rule, the portfolio with an expected return of 0.54% has a CVaR of 4.22%. In comparison, the next to last row of Table 5 indicates that in the presence of the Volcker rule, the portfolio with this expected return has a CVaR of 9.89%. Hence, when the required expected return is 0.54%, the presence of the Volcker rule more than doubles the CVaR of the minimum CVaR portfolio by increasing it by 5.67% \[= 9.89\% - 4.22\%\]. This result can be understood by noting that a reduction in the number
of available assets generally shifts the mean-CVaR frontier rightward for nearly all levels of expected return. Such a result is consistent with Thakor (2012) who recognizes that the Volcker rule will make bank risk management less efficient by constraining bank security holdings as noted earlier. As also noted earlier, Duffie (2012) and Thakor point out that it will adversely affect the liquidity of many securities. As a result, they conclude that the Volcker rule will negatively affect businesses by harming their ability to raise capital and increasing their cost of capital.

Of particular interest are the CVaRs of portfolios: (a) on the mean-CVaR frontier; and (b) with maximum efficiency losses. Like in the case where the Volcker rule is absent, we focus for brevity on the case where variable-bound constraints are imposed. The CVaRs of portfolios on the mean-CVaR frontier range from $-0.05\%$ to $20.17\%$ with the median CVaR being $6.35\%$. The CVaRs of portfolios with maximum efficiency losses also range from $-0.05\%$ to $20.17\%$, but the median CVaR is $7.65\%$.\textsuperscript{39} Hence, the median CVaR of portfolios with maximum efficiency losses exceeds the median CVaR of portfolios on the mean-CVaR frontier by $1.30\%$ [$= 7.65\% - 6.35\%$] in absolute terms, and $20.46\%$ [$= 1.30\%/6.35\%$] in relative terms.

Note that the largest and median CVaRs of both portfolios on the mean-CVaR frontier and portfolios with maximum efficiency losses in the presence of the Volcker rule are smaller than the ones in its absence. As noted earlier, the largest CVaR of portfolios on the mean-CVaR frontier (as well as of portfolios with maximum efficiency losses) is $20.17\%$ in the presence of Volcker rule and $63.73\%$ in its absence. Also, the median CVaR of portfolios on the mean-CVaR frontier is $6.35\%$ in the presence of Volcker rule and $8.49\%$ in its absence. Similarly, the median CVaR of portfolios with maximum efficiency losses is $7.65\%$ in the presence of Volcker rule and $8.49\%$ in its absence.

\textsuperscript{39}As in the absence of the Volcker rule, the minimum CVaR of portfolios with maximum efficiency losses (i.e., $-0.05\%$) coincides with the minimum CVaR of portfolios on the mean-CVaR frontier. Similarly, the maximum CVaR of portfolios with maximum efficiency losses (i.e., $20.17\%$) coincides with the maximum CVaR of portfolios on the mean-CVaR frontier; see footnote 33.
The extent to which the median CVaR of portfolios with maximum efficiency losses exceeds the median CVaR of portfolios on the mean-CVaR frontier is also smaller in the presence of the Volcker rule. As noted earlier, the former median exceeds the latter by 1.30% in absolute terms and by 20.46% in relative terms when the Volcker rule is present. In comparison, the corresponding values in the absence of the Volcker rule are, respectively, 1.90% and 22.42%.

In sum, we find that when the Volcker rule is present, constraints (11) and (12) still allow the selection of portfolios with substantive efficiency losses, albeit to a lesser extent than when it is absent.\textsuperscript{40} This finding suggests that the Volcker rule is beneficial in partially mitigating the ineffectiveness of the Basel framework.

4. Additional robustness checks

Next, we further assess the robustness of our base-case findings by examining three additional cases. These cases consider a larger number of: (1) stress testing events; (2) assets; and (3) both stress testing events and assets. Since the base-case efficiency losses with variable-bound constraints are smaller than those with fixed-bound constraints, we only present the results with the former constraints.

4.1. Increasing the number of stress testing events

Consider a larger number of stress testing events along with the two original sets of seventeen assets (i.e., Treasury bills, Treasury bonds, Agency bonds, and Fama-French equity portfolios) and eleven assets (i.e., Treasury bills, Treasury bonds, and Agency bonds).

\textsuperscript{40}We confirm this finding by considering two additional cases (available upon request). In the first case, we exclude the stress testing events where all the assets involving bonds have positive returns. Accordingly, this case considers only two (instead of four) stress testing events: U.S. stock market crash of 1987 and U.S. interest rate hike of 1994; see the last four rows of Table 1. In the second case, we use the period January 1994–December 1994 (instead of January 2008–December 2008) to determine stressed VaR. Note that the assets involving bonds have all negative returns over this period (as the third to last row of Table 1 suggests).
Specifically, suppose that eight stress testing events are used. In addition to the four stress testing events used earlier, the set of eight stress testing events includes: (i) Russian crisis of 1998; (ii) LTCM collapse of 1998; (iii) dot-com slowdown of 2001–02; and (iv) Greek crisis of 2010. Column (2) of Fig. 4 provides a box plot of maximum efficiency losses with eight stress testing events and seventeen assets. Comparing columns (1) and (2), note that the losses are smaller than those with four stress testing events and seventeen assets, but they are still sizeable.\footnote{We do not claim that the Basel framework is equally ineffective in preventing banks from taking substantive risk in their trading portfolios without capital requirement penalties in all settings. Our point is that there exist many plausible settings in which this framework is indeed ineffective in doing so.} Similarly, column (2) of Fig. 5 provides a box plot of maximum efficiency losses with eight stress testing events and eleven assets. Comparing columns (1) and (2) of this figure, notice that the losses are also smaller than those with four stress testing events and eleven assets.\footnote{More generally, an increase in the number of stress testing events leads to either smaller or the same efficiency losses. This result can be understood by noting that such an increase either reduces the size of the set of portfolios that meet the constraints on the losses in stress testing events (in conjunction with the constraint on the sum of VaR and stressed VaR) or does not affect it. Since efficiency losses are non-decreasing in the size of this set, an increase in the number of stress testing events leads to either smaller or the same efficiency losses.} Importantly, while the losses with eleven assets are smaller than those with seventeen assets, they are not close to zero; see column (2) of Figs. 4 and 5, noting that the $y$-axes of the two figures use different scales. Column (2) of Fig. 6 shows a box plot of relative reductions in efficiency losses arising from using eleven assets instead of seventeen assets with eight stress testing events. Comparing columns (1) and (2) of this figure, note that such reductions are generally larger than those with four stress testing events. However, the first quartile is still about 50%, whereas the median is below 75%. In sum, the use of a larger number of stress testing events is beneficial but does not substantively alter our base-case findings that: (1) the Basel framework is ineffective in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties; and (2) the Volcker rule only partially mitigates the ineffectiveness of the Basel framework.
4.2. Increasing the number of assets

Consider a larger number of assets along with the original four stress testing events. As before, suppose that Treasury bills are available. However, we now use Treasury and Agency bonds with six maturity ranges (1–3, 3–5, 5–7, 7–10, 10–15, and 15+ years) instead of five. Furthermore, we now use the ten size-based Fama-French equity portfolios instead of the six book-to-market/size-based ones. Hence, the number of assets increases from seventeen to twenty-three when Treasury bills, Treasury bonds, Agency bonds, and the Fama-French equity portfolios are available (i.e., when the Volcker rule is absent). Additionally, it increases from eleven to thirteen when only Treasury bills, Treasury bonds, and Agency bonds are available (i.e., when the Volcker rule is present). Column (3) of Fig. 4 provides a box plot of maximum efficiency losses with twenty-three assets and four stress testing events. While the median loss is smaller than that with seventeen assets and four stress testing events, the upper quartile and highest losses are larger; see columns (3) and (1). Similarly, column (3) of Fig. 5 provides a box plot of maximum efficiency losses with thirteen assets and four stress testing events. Note that the median loss is comparable to that with eleven assets and four stress testing events, but the upper quartile and highest losses are larger; again, see columns (3) and (1). Moreover, while the median loss with thirteen assets and four stress testing events is comparable to that with twenty-three assets and four stress testing events, the upper quartile and highest losses are considerably smaller with thirteen assets and four stress testing events; see column (3) of Figs. 4 and 5, again noting that the y-axes of the two figures use different scales. Importantly, the losses with thirteen assets and four stress testing events are not close to zero. Column (3) of Fig. 6 shows a box plot of relative

\[43\text{More generally, an increase in the number of assets leads to either smaller, the same, or larger efficiency losses. This result can be understood by noting that such an increase has two effects: (1) it generally shifts the mean-CVaR frontier leftward for nearly all levels of expected return; and (2) it possibly reduces the size of the variable bounds (see Eqs (13) and (14)). Since efficiency losses are determined relative to the mean-CVaR frontier, the first effect generally increases the size of efficiency losses. In contrast, since the use of smaller bounds corresponds to tighter constraints, the second effect generally decreases the size of efficiency losses. Hence, an increase in the number of assets leads to either smaller, the same, or larger efficiency losses.}\]
reductions in efficiency losses arising from using thirteen assets instead of twenty-three assets with four stress testing events. Comparing columns (1) and (3) of this figure, note that such reductions are similar to those arising from using eleven assets instead of seventeen assets and four stress testing events. In sum, the use of a larger number of assets also confirms our base-case findings.

4.3. Increasing the numbers of stress testing events and assets

Consider larger numbers of both stress testing events and assets. Column (4) of Fig. 4 shows a box plot of maximum efficiency losses with eight stress testing events and twenty-three assets. While the median loss is smaller than that with seventeen assets and four stress testing events shown in column (1), the upper quartile and highest losses are larger. Similarly, column (4) of Fig. 5 provides a box plot of maximum efficiency losses with eight stress testing events and thirteen assets. Note that the losses are smaller than those with four stress testing events and eleven assets shown in column (1). While losses with thirteen assets and eight stress testing events are smaller than losses with twenty-three assets and eight stress testing events, the former losses are not close to zero; see column (4) of Figs. 4 and 5 once more noting that the y-axes of the two figures use different scales. Column (4) of Fig. 6 shows a box plot of relative reductions in efficiency losses arising from using thirteen assets instead of twenty-three assets with eight stress testing events. Comparing columns (1) and (4) of this figure, note that such reductions are possibly larger than those arising from

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44 Unlike previous cases, there are a few outliers in the relative reductions in efficiency losses in the case associated with column (3). Here, an outlier is defined as a value that is above (below) the upper (lower) quartile by an amount that exceeds 1.5 times the size of the interquartile range. Column (3) reports the results without outliers. All of these outliers involve smaller (and negative) relative reductions in efficiency losses. A negative relative reduction in efficiency loss for a given level of expected return indicates that the presence of the Volcker rule increases the size of the efficiency loss at this level of expected return. Hence, the inclusion of the outliers would strengthen the result that the Volcker rule only partially mitigates the ineffectiveness of the Basel framework.

45 More generally, an increase in the number of both stress testing events and assets leads to either smaller, the same, or larger efficiency losses. This result can be understood by noting two observations that were made earlier: (a) an increase in the number of stress testing events leads to either smaller or the same efficiency losses; and (b) an increase in the number of assets leads to either smaller, the same, or larger efficiency losses.
using eleven assets instead of seventeen assets and four stress testing events. However, such reductions can also be smaller (see, for example, the lowest value of the reduction). In sum, the use of larger numbers of stress testing events and assets also confirms our base-case findings.

5. Alternatives to the Basel framework and impact of the Volcker rule

Since our results suggest that the Basel framework is ineffective and the Volcker rule only partially mitigates this ineffectiveness, we present alternatives to the Basel framework next.

5.1. Alternatives to the Basel framework

Although we do not seek to be exhaustive in considering all possible alternatives to the Basel framework, we now briefly discuss three them. Each of these alternatives involves a different approach for determining the minimum capital requirements associated with trading portfolios. First, regulators could base such requirements on CVaR instead of basing them on the sum of VaR and stressed VaR. Second, regulators could base minimum capital requirements on VaRs at multiple confidence levels (e.g., 99% and 99.5%). Third, regulators could base minimum capital requirements on VaR at a higher confidence level (e.g., 99.5%)

Like in the case associated to column (3), there are a few outliers in the relative reductions in efficiency losses in the case associated with column (4). As in column (3), column (4) reports the results without outliers; see footnote 44. For each of the three robustness cases (i.e., larger number of stress testing events, larger number of assets, and larger numbers of stress testing events and assets), there is statistical significance to the result that the distribution (median) of losses with Treasury bills, Treasury bonds, Agency bonds, and Fama-French equity portfolios differs from that with Treasury bills, Treasury bonds, and Agency bonds with a single exception. Specifically, in the case of a larger number of assets, we do not find statistical evidence that the median of losses with Treasury bills, Treasury bonds, Agency bonds, and Fama-French equity portfolios differs from that with Treasury bills, Treasury bonds, and Agency bonds; see column (3) of Figs. 4 and 5, again noting that the y-axes of the two figures use different scales. As before, we assess the statistical significance of the differences between the distributions and medians of losses by using, respectively, the two-sample Kolmogorov-Smirnov and Wilcoxon rank sum tests (available upon request).

Motivation for this alternative can be found in work reviewed earlier (see footnote 6). The Basel Committee on Banking Supervision (2012) presents a preliminary proposal to use stressed CVaR to determine the minimum capital requirements associated with trading portfolios. The Basel Committee on Banking Supervision sought comments on this proposal from the public until September 7, 2012. After the Basel Committee on Banking Supervision reviews such comments, it intends to release for comment a more detailed proposal to amend the Basel framework for trading portfolios.

Motivation for this alternative can be found in Alexander, Baptista, and Yan (2012). Indeed, they find that risk management systems based on multiple VaR constraints are more effective in controlling tail risk than systems based on a single VaR constraint.
instead of 99%). For each of these alternatives, regulators could still require the use of a rigorous stress testing program. Importantly, the adoption of any one of them would mitigate the ineffectiveness of the Basel framework. For example, a system based on a framework that properly uses CVaR to set minimum capital requirements would (by construction) be fully effective in preventing the selection of portfolios with substantive tail risk. Hence, this framework would (again by construction) fully mitigate the ineffectiveness of the Basel framework. Also, while a system based on a framework that properly uses either VaRs at multiple confidence levels or VaR at a confidence level higher than 99% to set minimum capital requirements might not be fully effective in preventing the selection of portfolios with substantive tail risk, it would be more effective than one based on the Basel framework. Hence, the former framework would at least partially mitigate the ineffectiveness of the latter.

5.2. Impact of the Volcker rule

Next, we briefly examine the impact of the Volcker rule if any one of the aforementioned alternatives to the Basel framework is adopted. If any one of them is in place, then the Volcker rule would have both benefits and costs (like in the case where the Basel framework is in place as discussed earlier). First, consider the benefits of the Volcker rule. Suppose that the alternative is fully effective in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties. By definition, such an alternative would force banks to select trading portfolios on the mean-CVaR frontier. Since the Volcker rule precludes the selection of portfolios that involve stocks, the maximum CVaR of the trading portfolios on the mean-CVaR frontier in its presence is notably smaller than that in its absence. Hence, the Volcker rule would have the benefit of precluding the selection

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50 Motivation for this alternative can also be found in Alexander, Baptista, and Yan (2012). Under certain conditions, they find that the use of VaR is more effective in controlling tail risk if it uses a higher confidence level.
of trading portfolios with relatively large CVaRs. Suppose now that the alternative is not fully effective in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties. By definition, such an alternative would allow banks to select trading portfolios that lie away from the mean-CVaR frontier. Since some of these portfolios involve stocks, the Volcker rule would have the benefit of precluding their selection.

Second, consider the costs of the Volcker rule. As noted earlier, Duffie (2012) and Thakor (2012) point out that it will adversely affect the liquidity of many securities. As a result, they conclude that the Volcker rule will negatively affect businesses by harming their ability to raise capital and increasing their cost of capital. Also, Thakor recognizes that by constraining bank security holdings, this rule will make bank risk management less efficient.

Assuming that the benefits of the Volcker rule significantly exceed its costs, the adoption of this rule would be valuable even if the Basel framework is replaced by any one of the aforementioned alternative frameworks. However, to the extent that these alternative frameworks mitigate the ineffectiveness of the Basel framework, the benefits of the Volcker rule would be smaller.

6. Conclusion

Banks around the world suffered huge trading losses in the recent financial crisis. Such losses suggest that these banks take substantive tail risk within their trading portfolios. In response to the crisis, the Basel Committee on Banking Supervision (2011) provides a revised framework for attempting to control this risk. Moreover, the Dodd-Frank Wall Street Reform and Consumer Protection Act (2010) imposes certain restrictions on the composition of the trading portfolios of U.S. banks through the so-called Volcker rule. Our paper assesses the effectiveness of the Basel framework and Volcker rule in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties. We
find that the Basel framework is ineffective in preventing banks from doing so. However, we find that the Volcker rule partially mitigates the ineffectiveness of the Basel framework. These findings suggest that while the Volcker rule is beneficial in supplementing the Basel framework, it does not fully address the ineffectiveness of such a framework.

We also discuss alternatives to the Basel framework. Instead of using VaR and stressed VaR to determine the minimum capital requirements for trading portfolios, we suggest the use of either: (1) CVaR; (2) VaRs at multiple confidence levels; or (3) VaR at a confidence level higher than 99%. The adoption of any one of these alternatives would mitigate the ineffectiveness of the Basel framework. To the extent that such alternative frameworks do so, the benefits of the Volcker rule would be smaller.

We emphasize that the adoption of the Volcker rule has costs. Duffie (2012) and Thakor (2012) point out that it will adversely affect the liquidity of many securities. As a result, they argue that the Volcker rule will negatively affect businesses by harming their ability to raise capital and increasing their cost of capital. Furthermore, Thakor notes that by constraining bank security holdings, this rule will make bank risk management less efficient.
References


Table 1: Summary statistics on asset returns

Using daily data during 1987–2011, this table presents summary statistics on the returns of the following assets: (a) Treasury bills; (b) Treasury bonds with five maturity ranges (1–3, 3–5, 5–10, 10–15, and 15+ years); (c) Agency bonds with the same maturity ranges; and (d) the six size/book-to-market-based Fama-French equity portfolios. These portfolios result from sorting stocks along the dimensions of: (i) market capitalization (small and big); and (ii) book-to-market ratio (low, intermediate, and high). Returns on Treasury bills and Fama-French portfolios are obtained from Kenneth French’s website. Returns on Treasury and Agency bonds are obtained from Bloomberg by using the Bank of America Merrill Lynch Treasury and Agency bond indices, respectively. In estimating VaR, CVaR, and stressed VaR, we use a confidence level of 99%. Estimates of stressed VaR use the period January 2008–December 2008. Also presented are the asset returns in four stress testing events: (1) U.S. stock market crash of 1987 (October 14–19); (2) U.S. interest rate hike of 1994 (January 31–December 13); (3) terrorist attacks of 2001 (September 11–17); and (4) subprime crisis of 2007–09 (October 1, 2007–February 27, 2009). With the exception of such returns, the reported statistics are scaled to a period of ten trading days. In doing so, we multiply the average daily return by ten and the remaining statistics by the square root of ten. All numbers are reported in percentage points.

<table>
<thead>
<tr>
<th></th>
<th>Treasury bonds</th>
<th>Agency bonds</th>
<th>Fama-French equity portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Treasury</td>
<td>Agency</td>
<td></td>
</tr>
<tr>
<td></td>
<td>bills</td>
<td>Maturity</td>
<td>Maturity</td>
</tr>
<tr>
<td></td>
<td>1–3</td>
<td>(years)</td>
<td>3–5</td>
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<tr>
<td>Average return</td>
<td>0.15</td>
<td>0.21</td>
<td>0.30</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.00</td>
<td>0.32</td>
<td>0.73</td>
</tr>
<tr>
<td>VaR</td>
<td>-0.05</td>
<td>0.78</td>
<td>1.94</td>
</tr>
<tr>
<td>CVaR</td>
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<td>1.10</td>
<td>2.46</td>
</tr>
<tr>
<td>Stressed VaR</td>
<td>-0.05</td>
<td>1.44</td>
<td>2.91</td>
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Stress testing events:

<table>
<thead>
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<th></th>
<th>Returns in stress testing events</th>
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</thead>
<tbody>
<tr>
<td>U.S. stock market crash of 1987</td>
<td>-0.11 -0.09 -0.44 -1.09 -2.20 -2.49 -0.19 -0.76 -1.59 -1.70 -2.60</td>
</tr>
<tr>
<td>Terrorist attacks of 2001</td>
<td>0.02 0.94 1.64 1.79 1.45 1.01 0.79 1.32 1.60 1.58 0.89</td>
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</table>

All numbers are reported in percentage points.
This table examines the effectiveness of the Basel framework in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties when the Volcker rule is absent. There are seventeen assets: (a) Treasury bills; (b) Treasury bonds with five maturity ranges (1–3, 3–5, 5–10, 10–15, and 15+ years); (c) Agency bonds with the same maturity ranges; and (d) six size/book-to-market-based Fama-French equity portfolios. Also, there are four stress testing events: (i) U.S. stock market crash of 1987; (ii) U.S. interest rate hike of 1994; (iii) terrorist attacks of 2001; and (iv) subprime crisis of 2007–09. The first four columns use fixed bounds that do not depend on required expected return $E$. The fixed bound on the sum of VaR and stressed VaR is $U^{V+\nabla} = 4\%$ or $20\%$, whereas the fixed bound on the loss in stress testing event $k$ is $U^{L_k} = 4\%$ or $20\%$, where $k = 1, \ldots, 4$. The last column uses variable bounds that depend on required expected return $E$. The variable bound on the sum of VaR and stressed VaR is $U^{V+\nabla}_E$ as defined by Eq. (12), whereas the variable bound on the loss in stress testing event $k$ is $U^{L_k}_E$ as defined by Eq. (13), where $k = 1, \ldots, 4$. A confidence level of 99% is used to compute VaR, stressed VaR, and efficiency losses. We scale the reported fixed bounds on the sum of VaR and stressed VaR, efficiency losses, and expected returns to a period of ten trading days. In doing so, we multiply: (a) daily bounds and daily efficiency losses by the square root of ten; and (b) daily expected returns by ten. In contrast, the reported fixed bounds on the losses in stress testing events apply to the periods of time capturing the events as defined in Table 1. Efficiency losses, relative efficiency losses, and expected returns are reported in percentage points.

<table>
<thead>
<tr>
<th>Efficiency loss:</th>
<th>$U^{V+\nabla}$</th>
<th>$U^{V+\nabla}_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4%</td>
<td>20%</td>
</tr>
<tr>
<td>Efficiency loss:</td>
<td></td>
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<tr>
<td>Average</td>
<td>2.55</td>
<td>2.56</td>
</tr>
<tr>
<td>Largest</td>
<td>3.83</td>
<td>3.85</td>
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<td>Relative efficiency loss:</td>
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<td></td>
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<tr>
<td>Average</td>
<td>126.07</td>
<td>126.40</td>
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<tr>
<td>Largest</td>
<td>267.67</td>
<td>267.30</td>
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<td>Euclidean distance:</td>
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<td></td>
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<tr>
<td>Average</td>
<td>45.03</td>
<td>46.35</td>
</tr>
<tr>
<td>Largest</td>
<td>58.58</td>
<td>59.62</td>
</tr>
<tr>
<td>Maximum feasible expected return</td>
<td>0.41</td>
<td>0.41</td>
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</table>
Table 3: Return and risk statistics for selected portfolios when the Volcker rule is absent

Fix the two levels of expected return, $E_{33}$ and $E_{67}$, that are, respectively, 33% and 67% of the way between the risk-free return ($E$) and the maximum feasible expected return ($E$). This table presents return and risk statistics for: (i) the portfolios on the mean-CVaR frontier when such levels of expected return are required; and (ii) the portfolios with maximum efficiency losses when variable-bound constraints are imposed and the same levels of expected return are required. These constraints limit the sum of VaR and stressed VaR as well as losses in stress testing events; see Eqs. (11)–(14). There are seventeen assets: (a) Treasury bills; (b) Treasury bonds with five maturity ranges (1–3, 3–5, 5–10, 10–15, and 15+ years); (c) Agency bonds with the same maturity ranges; and (d) the six size/book-to-market-based Fama-French equity portfolios. Also, there are four stress testing events: (i) U.S. stock market crash of 1987; (ii) U.S. interest rate hike of 1994; (iii) terrorist attacks of 2001; and (iv) subprime crisis of 2007–09. With the exception of losses in stress testing events and relative efficiency losses, the reported statistics are scaled to a period of ten trading days. In doing so, we multiply the expected daily return by ten and the remaining statistics by the square root of ten. A confidence level of 99% is used to estimate VaR, stressed VaR, and CVaR. All numbers are reported in percentage points.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected return</th>
<th>VaR</th>
<th>Stressed VaR</th>
<th>Sum of VaR and stressed VaR</th>
<th>Losses in stress testing events</th>
<th>CVaR</th>
<th>Efficiency loss</th>
<th>Relative efficiency loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{33}$:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On mean-CVaR frontier</td>
<td>0.54</td>
<td>3.33</td>
<td>8.09</td>
<td>11.42</td>
<td>3.24</td>
<td>5.38</td>
<td>-1.37</td>
<td>-9.49</td>
</tr>
<tr>
<td>With maximum efficiency loss</td>
<td>0.54</td>
<td>5.02</td>
<td>6.40</td>
<td>11.42</td>
<td>3.24</td>
<td>5.36</td>
<td>-2.47</td>
<td>-13.64</td>
</tr>
<tr>
<td>$E_{67}$:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On mean-CVaR frontier</td>
<td>0.95</td>
<td>10.52</td>
<td>18.51</td>
<td>29.03</td>
<td>13.59</td>
<td>20.52</td>
<td>3.00</td>
<td>18.85</td>
</tr>
<tr>
<td>With maximum efficiency loss</td>
<td>0.95</td>
<td>11.85</td>
<td>17.17</td>
<td>29.03</td>
<td>7.10</td>
<td>20.52</td>
<td>2.82</td>
<td>16.90</td>
</tr>
</tbody>
</table>
This table examines the effectiveness of the Basel framework in preventing banks from taking substantive tail risk in their trading portfolios without capital requirement penalties when the Volcker rule is present. There are eleven assets: (a) Treasury bills; (b) Treasury bonds with five maturity ranges (1–3, 3–5, 5–10, 10–15, and 15+ years); and (c) Agency bonds with the same maturity ranges. Also, there are four stress testing events: (i) U.S. stock market crash of 1987; (ii) U.S. interest rate hike of 1994; (iii) terrorist attacks of 2001; and (iv) subprime crisis of 2007–09. The first four columns use fixed bounds that do not depend on required expected return $E$. The fixed bound on the sum of VaR and stressed VaR is $U^{V+\nabla} = 4\%$ or $20\%$, whereas the fixed bound on the loss in stress testing event $k$ is $U^{L_k} = 4\%$ or $20\%$, where $k = 1, \ldots, 4$. The last column uses variable bounds that depend on required expected return $E$. The variable bound on the sum of VaR and stressed VaR is $U^{V+\nabla}_E$ as defined by Eq. (12), whereas the variable bound on the loss in stress testing event $k$ is $U^{L_k}_E$ as defined by Eq. (13), where $k = 1, \ldots, 4$. A confidence level of $99\%$ is used to compute VaR, stressed VaR, and efficiency losses. We scale the reported fixed bounds on the sum of VaR and stressed VaR, efficiency losses, and expected returns to a period of ten trading days. In doing so, we multiply: (a) daily bounds and daily efficiency losses by the square root of ten; and (b) daily expected returns by ten. In contrast, the reported fixed bounds on the losses in stress testing events apply to the periods of time capturing the events as defined in Table 1. Efficiency losses, relative efficiency losses, and expected returns are reported in percentage points.

<table>
<thead>
<tr>
<th></th>
<th>$U^{V+\nabla}$</th>
<th>$U^{V+\nabla}_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4%</td>
<td>20%</td>
</tr>
<tr>
<td>$U^{L_k}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4%</td>
<td>1.89</td>
<td>1.90</td>
</tr>
<tr>
<td>20%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Largest</td>
<td>3.07</td>
<td>3.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Efficiency loss:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>77.53</td>
<td>77.82</td>
</tr>
<tr>
<td>Largest</td>
<td>179.34</td>
<td>179.49</td>
</tr>
<tr>
<td>Relative efficiency loss:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>59.89</td>
<td>61.05</td>
</tr>
<tr>
<td>Largest</td>
<td>76.18</td>
<td>80.40</td>
</tr>
<tr>
<td>Euclidean distance:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.29</td>
<td>0.29</td>
</tr>
</tbody>
</table>
Fix the two levels of expected return, $E_{33}$ and $E_{67}$, that are, respectively, 33% and 67% of the way between the risk-free return ($E_0$) and the maximum feasible expected return ($E_1$). This table presents return and risk statistics for: (i) the portfolios on the mean-CVaR frontier when such levels of expected return are required; and (ii) the portfolios with maximum efficiency losses when variable-bound constraints are imposed and the same levels of expected return are required. These constraints limit the sum of VaR and stressed VaR as well as losses in stress testing events; see Eqs. (11)–(14). There are eleven assets: (a) Treasury bills; (b) Treasury bonds with five maturity ranges (1–3, 3–5, 5–10, 10–15, and 15+ years); and (c) Agency bonds with the same maturity ranges. Also, there are four stress testing events: (i) U.S. stock market crash of 1987; (ii) U.S. interest rate hike of 1994; (iii) terrorist attacks of 2001; and (iv) subprime crisis of 2007–09. With the exception of losses in stress testing events and relative efficiency losses, the reported statistics are scaled to a period of ten trading days. In doing so, we multiply the expected daily return by ten and the remaining statistics by the square root of ten. A confidence level of 99% is used to estimate VaR, stressed VaR, and CVaR. All numbers are reported in percentage points.

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>$E_{33}$:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On mean-CVaR frontier</td>
<td>0.34</td>
<td>2.59</td>
<td>4.85</td>
<td>7.44</td>
<td>1.94</td>
<td>6.34</td>
<td>-1.81</td>
<td>-14.95</td>
<td>3.45</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>With maximum efficiency loss</td>
<td>0.34</td>
<td>3.26</td>
<td>4.18</td>
<td>7.44</td>
<td>1.38</td>
<td>6.34</td>
<td>-1.89</td>
<td>-18.65</td>
<td>4.19</td>
<td>0.74</td>
<td>21.40</td>
</tr>
<tr>
<td>$E_{67}$:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On mean-CVaR frontier</td>
<td>0.54</td>
<td>8.07</td>
<td>11.77</td>
<td>19.84</td>
<td>5.61</td>
<td>16.56</td>
<td>-2.13</td>
<td>-21.04</td>
<td>9.89</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>With maximum efficiency loss</td>
<td>0.54</td>
<td>8.33</td>
<td>11.51</td>
<td>19.84</td>
<td>5.04</td>
<td>16.56</td>
<td>-2.13</td>
<td>-24.31</td>
<td>10.72</td>
<td>0.83</td>
<td>8.37</td>
</tr>
</tbody>
</table>
Table 6: Reduction in efficiency losses arising from the Volcker rule

This table assesses the reduction in efficiency losses arising from the presence of the Volcker rule relative to the case where it is absent. When the Volcker rule is absent, there are seventeen assets: (a) Treasury bills; (b) Treasury bonds with five maturity ranges (1–3, 3–5, 5–10, 10–15, and 15+ years); (c) Agency bonds with the same maturity ranges; and (d) six size/book-to-market-based Fama-French equity portfolios. When the Volcker rule is present, there are only eleven assets: (a) Treasury bills; (b) Treasury bonds with the aforementioned five maturity ranges; and (c) Agency bonds with these maturity ranges. Regardless of whether the Volcker rule is absent or present, there are four stress testing events: (i) U.S. stock market crash of 1987; (ii) U.S. interest rate hike of 1994; (iii) terrorist attacks of 2001; and (iv) subprime crisis of 2007–09. The first four columns use fixed bounds that do not depend on required expected return $E$. The fixed bound on the sum of VaR and stressed VaR is $U^{V+V} = 4\%$ or $20\%$, whereas the fixed bound on the loss in stress testing event $k$ is $U^{Lk} = 4\%$ or $20\%$, where $k = 1, \ldots, 4$. The last column uses variable bounds that depend on required expected return $E$. The variable bound on the sum of VaR and stressed VaR is $U^{V+V}_E$ as defined by Eq. (12), whereas the variable bound on the loss in stress testing event $k$ is $U^{Lk}_E$ as defined by Eq. (13), where $k = 1, \ldots, 4$. A confidence level of 99\% is used to compute VaR, stressed VaR, and efficiency losses. We scale the reported fixed bounds on the sum of VaR and stressed VaR as well as the reported reductions in efficiency losses to a period of ten trading days. In doing so, we multiply daily bounds and daily reductions in efficiency losses by the square root of ten. In contrast, the reported fixed bounds on the losses in stress testing events apply to the periods of time capturing the events as defined in Table 1. Reductions in efficiency losses and relative reductions in efficiency losses are reported in percentage points.

<table>
<thead>
<tr>
<th></th>
<th>$U^{V+V}$</th>
<th>$U^{V+V}_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>$U^{Lk}$</td>
<td>$U^{Lk}_E$</td>
</tr>
<tr>
<td>Reduction in efficiency loss:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>1.39</td>
<td>1.40</td>
</tr>
<tr>
<td>Largest</td>
<td>2.49</td>
<td>2.55</td>
</tr>
<tr>
<td>Relative reduction in efficiency loss:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>44.72</td>
<td>44.72</td>
</tr>
<tr>
<td>Largest</td>
<td>98.22</td>
<td>99.92</td>
</tr>
</tbody>
</table>
Step 1: Set constraints on asset weights

Step 2: Choose the confidence level and bounds used by constraints on the: (a) sum of VaR and stressed VaR; and (b) losses in stress testing events*

Step 3: Find the maximum feasible expected return $\bar{E}$ given constraints on the: (i) asset weights; (ii) sum of VaR and stressed VaR; and (iii) losses in stress testing events

Step 4: Calculate $\delta \equiv (\bar{E} - \bar{E})/100$, where $\bar{E}$ is the risk-free rate

Step 5: Create a grid of 101 expected returns:
$E_0 = \bar{E}; E_1 = \bar{E} + \delta; \ldots; E_{100} = \bar{E}$

Step 6: For each level of expected return $E_i$, find the maximum efficiency loss, and determine the average and largest efficiency losses

Step 7: For each level of expected return $E_i$, find the relative efficiency loss, and determine the average and largest relative efficiency losses

Step 8: For each level of expected return $E_i$ in the grid for the case where the Volcker rule is present, find the reduction in maximum efficiency loss, and determine the average and largest reductions in efficiency losses

Step 9: For each level of expected return $E_i$ in the grid for the case where the Volcker rule is present, find the relative reduction in maximum efficiency loss, and determine the average and largest relative reductions in efficiency losses

* The bounds are either: (a) fixed (do not depend on required expected return $E_i$); or (b) variable (depend on $E_i$). Historical simulation is used to estimate VaR, stressed VaR, and CVaR for all portfolios.
The curve represents portfolios on the mean-CVaR frontier in the case where the Volcker rule is absent (the case where the Volcker rule is present is similar). The minimum required expected return $E_0 = \bar{E}$ is assumed to be the risk-free rate. The maximum required expected return $E_{100} = \bar{E}$ is assumed to be the maximum feasible expected return in the presence of constraints on the: (a) asset weights; (b) sum of VaR and stressed VaR; and (c) losses in stress testing events. Point $p_{i}^{\text{max}}$ represents the portfolio that has an expected return of $E_i$, satisfies these constraints, and has maximum CVaR, denoted by $C_{i}^{\text{max}}$. Point $p_{i}^{\text{min}}$ represents the portfolio that has the same expected return, satisfies the asset weight constraints, and has minimum CVaR, denoted by $C_{i}^{\text{min}}$. Suppose that the required expected return is $E_i$. Then, the maximum efficiency loss is $M_i \equiv C_{i}^{\text{max}} - C_{i}^{\text{min}}$, whereas the relative efficiency loss is $\text{REL}_i \equiv M_i / C_{i}^{\text{min}}$.
Fig. 3: Determining reductions in maximum efficiency losses

The solid curve represents portfolios on the mean-CVaR frontier when the Volcker rule is absent. Similarly, the dashed curve represents portfolios on the mean-CVaR frontier when the Volcker rule is present. Both curves consider the levels of expected return between the minimum required expected return and the maximum feasible expected return when the Volcker rule is present. Consider the case where the Volcker rule is absent. Point $p_{i,\text{max,ab}}$ represents the portfolio that has an expected return of $E_i$, satisfies constraints on the: (i) asset weights; (ii) sum of VaR and stressed VaR; and (iii) losses in stress testing events, and has maximum CVaR, denoted by $C_{i,\text{max,ab}}$. Point $p_{i,\text{min,ab}}$ represents the portfolio that has the same expected return, satisfies the asset weight constraints, and has minimum CVaR, denoted by $C_{i,\text{min,ab}}$. The maximum efficiency loss is $M_i^{\text{ab}} \equiv C_{i,\text{max,ab}} - C_{i,\text{min,ab}}$. Consider now the case where the Volcker rule is present. Points $p_{i,\text{min,pr}}$ and $p_{i,\text{max,pr}}$ represent, respectively, the portfolios with minimum and maximum CVaRs. The CVaRs of such portfolios are denoted by $C_{i,\text{min,pr}}$ and $C_{i,\text{max,pr}}$. The maximum efficiency loss is $M_i^{\text{pr}} \equiv C_{i,\text{max,pr}} - C_{i,\text{min,pr}}$. Hence, the reduction in maximum efficiency loss is $RM_i \equiv M_i^{\text{ab}} - M_i^{\text{pr}}$, whereas the relative reduction in maximum efficiency loss is $RRM_i \equiv RM_i / M_i^{\text{ab}}$. 

maximum efficiency loss when the Volcker rule is present:
$M_i^{\text{pr}} \equiv C_{i,\text{max,pr}} - C_{i,\text{min,pr}}$

maximum efficiency loss when the Volcker rule is absent:
$M_i^{\text{ab}} \equiv C_{i,\text{max,ab}} - C_{i,\text{min,ab}}$

reduction in maximum efficiency loss:
$RM_i \equiv M_i^{\text{ab}} - M_i^{\text{pr}}$

relative reduction in maximum efficiency loss:
$RRM_i \equiv RM_i / M_i^{\text{ab}}$
This figure shows box plots of maximum efficiency losses when the Volcker rule is absent. All columns consider variable-bound constraints. The variable bound on the sum of VaR and stressed VaR is $U^{V+E}$ as defined by Eq. (12), whereas the variable bound on the loss in stress testing event $k$ is $U^{L_k}$ as defined by Eq. (13), where $k = 1, \ldots, K$. Here, $E$ denotes the required expected return, whereas $K$ denotes the number of stress testing events. In columns (1) and (2), seventeen assets are available: (a) Treasury bills; (b) Treasury bonds with five maturity ranges (1–3, 3–5, 5–10, 10–15, and 15+ years); (c) Agency bonds with the same maturity ranges; and (d) six size/book-to-market-based Fama-French equity portfolios. In columns (3) and (4), twenty-three assets are available: (a) Treasury bills; (b) Treasury bonds with six maturity ranges (1–3, 3–5, 5–7, 7–10, 10–15, and 15+ years); (c) Agency bonds with the same maturity ranges; and (d) ten size-based Fama-French equity portfolios. Columns (1) and (3) use four stress testing events (i.e., $K = 4$): (i) U.S. stock market crash of 1987; (ii) U.S. interest rate hike of 1994; (iii) terrorist attacks of 2001; and (iv) subprime crisis of 2007–09. Columns (2) and (4) use eight stress testing events (i.e., $K = 8$) with the four additional events being: (i) Russian crisis of 1998; (ii) LTCM collapse of 1998; (iii) dot-com slowdown of 2001–02; and (iv) Greek crisis of 2010. A confidence level of 99% is used to compute VaR, stressed VaR, and CVaR.
This figure shows box plots of maximum efficiency losses when the Volcker rule is present. All columns consider variable-bound constraints. The variable bound on the sum of VaR and stressed VaR is $U_{E}^{V+\overline{V}}$ as defined by Eq. (12), whereas the variable bound on the loss in stress testing event $k$ is $U_{E}^{L_{k}}$ as defined by Eq. (13), where $k = 1, ..., K$. Here, $E$ denotes the required expected return, whereas $K$ denotes the number of stress testing events. In columns (1) and (2), eleven assets are available: (a) Treasury bills; (b) Treasury bonds with five maturity ranges (1–3, 3–5, 5–10, 10–15, and 15+ years); and (c) Agency bonds with the same maturity ranges. In columns (3) and (4), thirteen assets are available: (a) Treasury bills; (b) Treasury bonds with six maturity ranges (1–3, 3–5, 5–7, 7–10, 10–15, and 15+ years); and (c) Agency bonds with the same maturity ranges. Columns (1) and (3) use four stress testing events (i.e., $K = 4$): (i) U.S. stock market crash of 1987; (ii) U.S. interest rate hike of 1994; (iii) terrorist attacks of 2001; and (vi) subprime crisis of 2007–09. Columns (2) and (4) use eight stress testing (i.e., $K = 8$) with the four additional events being: (v) Russian crisis of 1998; (vi) LTCM collapse of 1998; (vii) dot-com slowdown of 2001–02; and (viii) Greek crisis of 2010. A confidence level of 99% is used to compute VaR, stressed VaR, and CVaR.
Fig. 6: Box plots of relative reductions in efficiency losses arising from the Volcker rule

This figure shows box plots of relative reductions in efficiency losses arising from the Volcker rule. All columns consider variable-bound constraints. The variable bound on the sum of VaR and stressed VaR is $U^V + \overline{V}$ as defined by Eq. (12), whereas the variable bound on the loss in stress testing event $k$ is $U^E_L_k$ as defined by Eq. (13), where $k = 1, \ldots, K$. Here, $E$ denotes the required expected return, whereas $K$ denotes the number of stress testing events. Column (1) considers the relative reductions in efficiency losses arising from using eleven assets instead of seventeen assets with four stress testing events (i.e., $K = 4$). The set of eleven assets involves: (a) Treasury bills; (b) Treasury bonds with five maturity ranges (1–3, 3–5, 5–10, 10–15, and 15+ years); and (c) Agency bonds with the same maturity ranges. The set of seventeen assets involves these eleven assets and the six size/book-to-market-based Fama-French equity portfolios. The four stress testing events are: (i) U.S. stock market crash of 1987; (ii) U.S. interest rate hike of 1994; (iii) terrorist attacks of 2001; and (vi) subprime crisis of 2007–09. Column (2) is similar to column (1), but uses eight stress testing events (i.e., $K = 8$) instead of four. The four additional events are: (v) Russian crisis of 1998; (vi) LTCM collapse of 1998; (vii) dot-com slowdown of 2001–02; and (viii) Greek crisis of 2010. Column (3) considers the relative reductions in efficiency losses arising from using thirteen assets instead of twenty-three assets with four stress testing events. The set of thirteen assets involves: (a) Treasury bills; (b) Treasury bonds with six maturity ranges (1–3, 3–5, 5–7, 7–10, 10–15, and 15+ years); and (c) Agency bonds with the same maturity ranges. The set of twenty-three assets involves these thirteen assets and the ten size-based Fama-French equity portfolios. Column (4) is similar to column (3), but uses eight stress testing events instead of four. A confidence level of 99% is used to compute VaR, stressed VaR, and CVaR.