The Newsvendor Problem: Review and Directions for Future Research

Yan Qin, Ruoxuan Wang, Asoo J. Vakharia, Yuwen Chen, Michelle M. H. Seref
The Newsvendor Problem: Review and Directions for Future Research

Yan Qin\textsuperscript{1}
yan.qin@cba.ufl.edu

Ruoxuan Wang\textsuperscript{1}
rxwang@ufl.edu

Asoo J. Vakharia\textsuperscript{1}
asoov@ufl.edu

Yuwen Chen\textsuperscript{2}
yuwen@mail.uri.edu

Michelle MH Seref\textsuperscript{1}
michelle.hanna@cba.ufl.edu

January 2009

\textsuperscript{1} Department of Information Systems & Operations Management, Warrington College of Business Administration, University of Florida, Gainesville, FL 32611.
\textsuperscript{2} University of Rhode Island, Kingston, RI 02881.
The Newsvendor Problem: Review and Directions for Future Research

Abstract

In this paper, we review the contributions to date for analyzing the newsvendor problem. Our focus is on examining the specific extensions for analyzing this problem in the context of modeling customer demand, supplier costs, the buyer risk profile, and the selective newsvendor setting. Over and above summarizing the current literature, we also provide directions for future research in each area.

1 Introduction

The newsvendor problem is one of the classical problems in the literature on inventory management. Key insights stemming from an analysis of this problem have wide ranging implications for managing inventory decisions for organizations in, for example, the hospitality, airline, and fashion goods industries. A generic contemporary setting in which this problem could be framed is as follows. Consider a three-node supply chain consisting of a supplier, a buyer, and customers. At the beginning of a single period, the buyer is interested in determining an “optimal” stocking policy \( Q \) to satisfy total customer demand for a single product. This customer demand is assumed to be stochastic and characterized by a random variable \( x \) with density function \( f(x) \) and distribution function \( F(x) \). The quantity \( Q \) is purchased by the buyer from the supplier for a fixed price per unit \( v \). The supplier is assumed to operate with no capacity restrictions and zero lead time of supply and thus, an order placed by the buyer with the supplier at the beginning of a period is immediately filled. Sales of the product occur during (or at the end of) the single period and: (a) if \( Q \geq x \), then \( Q - x \) units which are left over at the end of the period are salvaged by the buyer for a per unit revenue of \( g \); and (b) if \( Q < x \), then \( x - Q \) units which represent “lost” sales cost the buyer \( B \) per unit. Assuming a fixed market price of \( p \), then the actual end of period profit for the buyer is:

\[
\Pi(Q, x) = \begin{cases} 
px - vQ + g(Q - x) & \text{if } Q \geq x \\
pQ - vQ - B(x - Q) & \text{if } Q < x
\end{cases}
\]

(1)

To eliminate trivial arbitrage opportunities for the buyer, it is generally assumed that \( g < v \).
Since this actual profit is not observable by the buyer at the beginning of the period, the
traditional approach to analyzing the problem is based on assuming a risk neutral buyer who
makes the optimal quantity decision at the beginning of the period by maximizing expected
profits. These profits are:

\[
E[\Pi(Q)] = \int_{-\infty}^{Q} [px - vQ + g(Q - x)]f(x)dx + \int_{Q}^{\infty} [pQ - vQ - B(x - Q)f(x)dx
\]

\[
= (p - g)\mu - (v - g)Q - (p - g + B)ES(Q) \tag{2}
\]

where \(\mu\) is the mean demand and \(ES(Q)\) represents the expected units short assuming
\(Q\) units are stocked and can be determined as \(\int_{Q}^{\infty} (x - Q)f(x)dx\). It is relatively easy to show
that equation (1) is strictly concave in \(Q\) (Silver, Pyke, and Peterson [45]) and hence, the
FOC are necessary and sufficient to determine the value of \(Q\) which maximizes equation (1).
This optimal order quantity \((Q^*)\) is set such that:

\[
F(Q^*) = \frac{p - v + B}{p - g + B} \tag{3}
\]

and the corresponding profit for the buyer is:

\[
E[\Pi(Q^*)] = (p - g)\mu - (p - g + B)\int_{Q^*}^{\infty} xf(x)dx \tag{4}
\]

The focus of this paper is on examining some of the key parametric assumptions which
drive this relatively straightforward analysis of the newsvendor problem. In essence, our
intent is on reviewing and examining how changes in the assumptions related to key pa-
rameters might moderate the current set of results. More specifically, this paper focuses on
reviewing prior work and developing extensions related to the following parameters.

1. Customer Demand: This is typically treated as an exogenous parameter in the prior
analysis. However, depending upon the the context, it could be argued that:

- Customer demand \(x\) and price \(p\) are interrelated;
- Customer demand \(x\) could be influenced by marketing effort \(e\) expended by the
  buyer; and
- Customer demand \(x\) could be influenced by the quantity stocked \(Q\) by the buyer.
2. Supplier Cost: A fixed cost per unit ($v$) quoted by the supplier is the current assumption in the newsvendor setting. However, current practice (see, for example, Burke, Carrillo, and Vakharia [10]) indicates suppliers tend to quote quantity discount schemes to generate larger order quantities from the buyer. Thus, it would be interesting to examine how alternative discounting schemes offered by the supplier might moderate the buyer’s optimal quantity decision.

3. Buyer Risk Profile: The assumption underlying the basic structural result for the newsvendor model is that the buyer is risk neutral and thus, would choose to optimize expected profits. Given that sourcing practice often indicates that buyers are risk averse (Burke and Vakharia [11]) and/or “risk” takers, we focus on how such risk profiles would moderate the sourcing decision in a newsvendor setting.

The remainder of this paper is organized as follows. In the next section, we start by analyzing the impact of alternative customer demand related changes in a newsvendor setting. Section 3 focuses on examining the impact of alternative supplier cost schemes; Section 4 describes how the risk profile of the supplier impacts the optimal decision making in this setting; and Section 5 describes contributions related to the selective newsvendor problem. Finally, implications and conclusions of this research are discussed in Section 6.

2 Customer Demand

2.1 Demand and Market Price

In a classic Newsvendor problem, selling price is considered as exogenous, over which the retailer has absolutely no control. This is true in a perfectly competitive market where sellers are mere price-takers. In practice, however, sellers have more or less the flexibility of adjusting the current selling price in order to increase or reduce the demand. Whitin [55], Mills [36] and Karlin and Carr [27] are three of the early publications that investigated the effects of different demand processes on the seller’s pricing and ordering decisions.

Whitin [55] provided necessary optimality conditions for two models, an EOQ model and a style goods model, under the assumption that mean demand is a linear function of price, i.e., $\mu_x = a + bp$, where $a$ and $b$ are some appropriate constants. Although Whitin
only provided the closed-form solution for a style goods model with a rectangular demand
distribution, he believed that formal solutions are possible as long as the price-dependent
demand distribution can be expressed as an explicit function of the expected demand.

Mills [36] assumed that the price-dependent demand is affected additively by a random
term, \( \epsilon \), independent of price, i.e. \( x = \mu_x(p) + \epsilon \). Different from deterministic demand
model that optimizes at the price which equates riskless marginal revenue and marginal
cost, the model under stochastic demand is optimized at the price which equates riskless
marginal revenue to marginal cost plus some generally non-zero term. He concluded that
the direction of change in the magnitude of the optimal price in the risky situation depends
on the shape of the marginal cost curve and shows that the optimal price cannot be higher
than it would be in the risk-less case for constant marginal cost. Karlin and Carr [27] showed
the reverse for the multiplicative model in which the stochastic demand \( x = \mu_x(p) \epsilon \).

The demand model employed in Mills [36] is referred to as the additive demand form and
the one in Karlin and Carr [27] as the multiplicative demand form. In what follows, we will
first derive some simple results for these two special demand-price relationships, and then
review of some key contributions on this topical area.

In the additive demand case, assume that \( x = \mu_x(p) + \epsilon \), where \( \mu_x(p) = a - bp \), \( a > 0 \), \( b > 0 \)
and the random term \( \epsilon \) follows a distribution of \( H(\cdot) \) with a mean of zero. The expected
profit function is now a bivariate function with price and order quantity as the decision
variables. Let \( c_o \) denote the overage cost \( v - g \) and \( c_u \) denote the underage cost \( p - v + B \).
The expected profit function can then be expressed as

\[
E[\Pi(p, Q)] = (p - v)\mu_x - c_o \cdot E[Q - x]^+ - c_u \cdot E[x - Q]^+ 
\]  

(5)

where

\[
E[Q - x]^+ = \int_0^{Q-a+bp} H(\epsilon)d\epsilon \\
E[x - Q]^+ = \int_{Q-a+bp}^{\infty} [1 - H(\epsilon)]d\epsilon 
\]

The expected profit function above simply says that expected profit equals riskless profit
minus expected cost. Next, let’s consider the partial derivatives of the expected profit
function with respect to \( Q \) and \( p \). Differentiating \( E[\Pi(p, Q)] \) with respect to \( Q \), we obtain

\[
\frac{\partial E[\Pi(p, Q)]}{\partial Q} = c_u - (c_o + c_u)H(Q - a + bp) 
\]
\[
\frac{\partial^2 E[\Pi(p, Q)]}{\partial Q^2} = -(c_o + c_u)h(Q - a + bp)
\]

Clearly, \( \frac{\partial^2 E[\Pi(p, Q)]}{\partial Q^2} \leq 0 \), which indicates that the expected profit function is concave in \( Q \) for a given \( p \). We can then solve for the optimal order quantity for a given \( p \) by setting the first order derivative equal to zero. It is quite straightforward to show that:

\[
H(Q_p^* - a + bp) = \frac{c_u}{c_o + c_u}
\]  

(6)

After substituting equation (2) into the expected profit function, we reduce the bivariate objective function into a univariate function of \( p \). Differentiating \( E[\Pi(p, Q_p^*)] \) with respect to \( p \) gives

\[
\frac{\partial E[\Pi(p, Q_p^*)]}{\partial p} = a - 2bp - ES(Q_p^*) + bv
\]

\[
\frac{\partial^2 E[\Pi(p, Q_p^*)]}{\partial p^2} = -2b + b \frac{v - g}{p - g + B}
\]

where \( ES(Q_p^*) = E[x - Q_p^*]^+ \). Due to the assumption that \( p > v > g \), \( \frac{\partial^2 E[\Pi(p, Q_p^*)]}{\partial p^2} < 0 \) for any eligible price. Again, by equating the first order derivative to zero, we obtain

\[
p_A^* = \frac{1}{2b} (a + bv - ES(Q_p^*))
\]  

(7)

In the deterministic case, \( p_D^* = \frac{a + bv}{2b} \). As \( ES(Q_p^*) \geq 0 \), \( p_A^* \leq p_D^* \) in the linear additive demand form. Moreover, we can see that no closed form expression for the optimal price can be obtained in this simple linear additive demand case. In general, the existence and/or uniqueness of \( Q_p^* \) for a given \( p \) cannot be guaranteed. Specific functional forms have to be assumed for both the mean demand and the distribution of the random term in order to ensure a well-behaved objective function.

Petruzzi and Dada [38] pointed out that a unique \( Q_p^* \) exists for a given \( p \) if the distribution of the random term \( \epsilon \) satisfies the inequality \( 2r^2(\epsilon) + \frac{\partial r(\epsilon)}{\partial \epsilon} > 0 \) for all possible values of \( \epsilon \), where \( r(\cdot) \equiv h(\cdot)/[1 - H(\cdot)] \) and \( a - b(v - 2B) + A > 0 \), where \( A \) is the minimum possible value of \( \epsilon \). The first inequality generalizes the conditions for which optimal solutions to single period newsvendor pricing problems can be identified analytically. Zhan and Shen [56] achieved the same results as above under the linear additive demand form and developed an iterative algorithm and a simulation based algorithm to find the optimal solution. Different from the
previous papers discussed so far, the authors treated this problem as a nonlinear system with two variables, instead of reducing the bivariate objective function into univariate.

In the multiplicative demand case, assume that \( x = \mu_x(p)\epsilon \), where \( \mu_x(p) = ap^{-b} \), \( a > 0, b > 1 \) and the random term \( \epsilon \) follows a distribution of \( H(\cdot) \) with variance of one. The expected profit function is again a bivariate function of price and order quantity.

\[
E[\Pi(p, Q)] = (p - v)\mu_x - c_o \cdot E[Q - x]^+ - c_u \cdot E[x - Q]^+ \tag{8}
\]

where

\[
E[Q - x]^+ = \int_0^{Qa^{-1}p^b} H(\epsilon)ap^{-b}d\epsilon \\
E[x - Q]^+ = \int_{Qa^{-1}p^b}^{+\infty} [1 - H(\epsilon)]ap^{-b}d\epsilon
\]

By applying the same procedure as in the additive case, we are able to show that the expected profit function is again concave in \( Q \) for a given \( p \) and that

\[
H(Q_p^*a^{-1}p^b) = \frac{c_u}{c_u + c_o} \tag{9}
\]

Differentiating \( E[\Pi(p, Q_p^*)] \) with respect to \( p \), we obtain

\[
\frac{\partial E[\Pi(p, Q_p^*)]}{\partial p} = [1 - b(1 - p^{-1}v)]ap^{-b}\mu_c + bp^{-1}(v - g) \cdot EL(Q_p^*) - \\
- [1 - bp^{-1}(p - v + B)] \cdot ES(Q_p^*)
\]

If \( \frac{\partial E[\Pi(p, Q_p^*)]}{\partial p} \leq 0 \), then \( p_M^* = \frac{bv}{b-1} + \Delta \), where

\[
\Delta = \frac{a^{-1}bp_M^{*b}[B \cdot ES(Q_{p_M^*}^*) + (v - g) \cdot EL(Q_{p_M^*}^*)]}{(b - 1)(\mu_c - a^{-1}p_M^{*b} \cdot ES(Q_{p_M^*}^*))} \tag{10}
\]

In the deterministic case, \( p_D^* = \frac{bv}{b-1} \). As \( \Lambda \geq 0 \), \( p_M^* \geq p_D^* \) in this specific multiplicative demand form. For the multiplicative demand form, Petruzzi and Dada [38] proposed that optimal solutions can be identified analytically if the distribution of the random term satisfies the inequality \( 2r^2(\epsilon) + \frac{\partial r(\epsilon)}{\partial \epsilon} > 0 \) for all possible values of \( \epsilon \) and \( b > 2 \). They also offered a possible explanation for the relative magnitudes of the optimal prices described above in terms of demand variance and demand coefficient of variation.
Lau and Lau [32] proposed several solution procedures for newsvendor problems with stochastic price-dependent demand and different optimization objectives, (1) maximization of the expected profit, and (2) maximization of the probability of achieving a target profit level. There are two cases of price-demand relationship considered in this paper, a homoscedastic linear regression model \( x = a - bp + \epsilon \) (Model A), where \( \epsilon \) is a normally distributed random term, and a model (Model B) constructed using a combination of statistical data analysis and experts’ subjective estimates. In case (A1), where demand assumes Model A and the optimization objective is to maximize the expected profit, the authors showed that the expected profit function is unimodal over both price and order quantity and thus the golden section method can be used to search for the optimal price. Effects of demand sensitivity to price and demand uncertainty are also investigated numerically in this case. The less sensitive that demand is to price, the higher the retail price and the expected profit. In case (B1), where demand is modeled by some four-parameter beta distribution, the authors showed that the expected profit function at \( Q^*(p) \) may not be unimodal over price \( p \) and thus some search procedure must be employed to find the optimal price. Search procedures are proposed for all other cases that intend to maximize the probability of achieving a target profit level except for case (A2) with no shortage cost. A counter-intuitive result was obtained for case (A2) that increased uncertainty may actually benefit the managerial objective, regardless of the existence of the shortage cost.

Polatoglu [39] investigated the joint pricing and ordering decisions under general demand uncertainty, aimed to reveal the fundamental properties independent of demand pattern. Unimodality of the expected profit function that traces the best price trajectory over the order-up-to decision was proved under the assumptions that the mean demand is a monotone decreasing function of price and that the risk-less revenue function, which is equal to price times mean demand, is uni-modal. Polatoglu [39] also approached the same results proven by Whitin [55], Mills [36] and Karlin and Carr [27], regarding the relative magnitudes of the optimal prices in additive and multiplicative models.

Federgruen and Heching [19] analyze multi-period price-dependent newsvendor problem when demands in consecutive periods are independent but the distributions are in accordance with general stochastic demand function. They assume that excess inventory is charged with holding cost and the excess demand is backlogged and charge with the same cost. In finite horizon and bi-directional price change case, they found that base stock list price policy
proposed in Porteus [40] is the optimal in each period such that the firm order extra items to the base level when the initial inventory is below that level or slash the price accordingly otherwise. The optimal price selected is nonincreasing of the initial inventory. If only markdowns are allowed, the base stock list price policy is slightly changed by the limitation.

Chen, Yan and Yao [15] is the only paper investigating the price-dependent newsvendor model in a competitive environment. The general demand is stochastic and price-dependent and the firm use price to compete for the demand. They prove and show the conditions for the existence of the pure-strategy Nash equilibrium and its uniqueness. Raising prices at any equilibrium of the game can increase the total system profit, but no firm would do it because it has optimized its profit given other firms’ pricing scheme. At the cooperative equilibrium, each self-interested firm has an incentive to lower its price to grab more demand. They conclude that competition leads to lower equilibrium price and higher stock levels in all firms.

Arcelus, Kumar and Srinivasan [5] studied the pricing and ordering policies of a profit-maximizing newsvendor that faces trade incentives offered by the manufacturer (i.e., wholesale price discount to the newsvendor and direct rebate to the end customers) and stochastic price-dependent demand. Let \((v, R)\) denote the manufacturer’s pricing/rebate policy, where \(v\) is the wholesale price and \(R\) is the direct rebate to end customers. Suppose the original wholesale price is \(v_0\). The newsvendor receives an offer \((v_0 - d, 0)\) under the price discount policy and \((v_0, R)\) under the direct rebate policy. No specific functional form is assumed for the deterministic component of the demand and both additive-error and multiplicative-error demands are considered in this paper. The authors derived the demand conditions under which the results obtained in Petruzzi and Dada [38] regarding the relative magnitude of the retail prices in risk-less and risky cases still hold in this setting. The effectiveness of the two trade incentives on the newsvendor’s decisions is then measured and contrasted by the pass-through ratio, defined as \(\delta = \frac{\partial p}{\partial v}\), under the price-discount policy, and the claw-back ratio, defined as \(\lambda = \frac{\partial p}{\partial R}\), under the direct-rebate policy. They showed analytically that the pass-through ratio is always greater than the claw-back ratio for any deterministic demand and that the claw-back ratio in the risky case is smaller than that in the risk-less case for demand forms with additive-error and linear deterministic component. It is worth noting that negative claw-back ratio is possible, that is, the newsvendor may offer a lower retail price to the end customers in spite of the rebate they’ve already got from the manufacturer.
However, no further analytical results are obtained concerning the impact of demand uncertainty on the newsvendor’s optimal order quantity in this setting. Numerical analysis is conducted under the assumption of uniformly distributed error term. Observations include the dominance of the pass-through ratio over the claw-back ratio in the risky case and the improved newsvendor’s expected profits under either trade incentive.

2.2 Demand and Marketing Effort

Customer demand can often be influenced by a number of marketing activities (e.g., advertising, sales calls, and in-store displays). Krauseburd, Narayanan and Raman [30] studied a model that incorporates such an aspect in the newsvendor scenario. Their setting is as follows. Let $e$ represent the marketing “effort” expended by the buyer and it is explicitly assumed that mean customer demand is a concave, non-decreasing function of $e$ (i.e., $\frac{\partial \mu(e)}{\partial e} \geq 0$) while variance in customer demand (i.e., $\sigma$) is not impacted by effort. In addition, let $c(e)$ represent the cost per unit of effort which is assumed to be convex in $e$. In this case, the optimal marketing effort allocation for the firm (i.e., $e^*$) can be determined by solving the following FOC:

$$
(p - v) \frac{\partial \mu(e)}{\partial e} - \frac{\partial c(e)}{\partial e} = 0 \tag{11}
$$

Once we know $e^*$, the buyer determines $Q^*$ such that $F(Q^*) = \frac{p - v + B}{p - g + B}$ except that the here the distribution function of demand is such that the mean is $\mu(e^*)$ and standard deviation is $\sigma^2$.

The analysis presented above could also be easily extended to a situation where marketing effort affects demand in a way that demand variance decreases as more effort is made in the selling season. Gerchak and Mossman [21] examined the effects of demand randomness on optimal order quantities and the associated expected costs by applying mean-preserving transformations to the demand variable. They considered the family of random variables

$$
x_\alpha = \alpha x + (1 - \alpha) \mu_x, \alpha \geq 0.
$$

$^2$For example, if demand is normally distributed with mean $\mu(e^*)$ and standard deviation $\sigma$, then $Q^* = \mu(e^*) + z\sigma$ where $z$ is the unit normal deviate such that $P(x \leq z) = \frac{p - v + B}{p - g + B}$.
Obviously, $E(x_\alpha) = \mu_x$ for all $\alpha$. We note that if $\alpha_1 > \alpha_2$, then $x_{\alpha_1}$ is more variable than $x_{\alpha_2}$ and using this property the authors are able to show that:

$$Q^*_\alpha = \alpha Q^* + (1 - \alpha)\mu_x$$

$$C_\alpha(Q^*_\alpha) = \alpha C(Q^*)$$

where $C(Q)$ represents the expected cost associated with an order quantity of $Q$.

To analyze the case in which demand uncertainty is affected by marketing effort, replace $\alpha$ with $\alpha(e)$ in the above three equations. Assume that $\alpha(e)$ is a monotone decreasing function of $e$ and that $\alpha(0) = 1, \alpha(+\infty) = 0$. The equation $\alpha(+\infty) = 0$ simply says that demand uncertainty may be eliminated by tremendous marketing effort, which is impossible in practice due to the unbearable cost of those marketing activities. Also assume that $c(0) = 0$.

After substituting and rearranging, we obtain

$$Q^*_{\alpha(e)} = \alpha(e)Q^* + (1 - \alpha(e))\mu_x$$

Consider the relative magnitudes of $Q^*_{\alpha(e)}$ and $Q^*$. As

$$\frac{\partial Q^*_{\alpha(e)}}{\partial e} = \frac{\partial \alpha(e)}{\partial e} (Q^* - \mu_x),$$

the optimal order quantity increases in marketing effort $e$ if and only if $Q^* < \mu_x$, i.e.,

$$F(\mu_x) > \frac{b+g+B}{b-g+B}.$$  

The maximum expected profit for a specific marketing effort $e$ can be expressed as

$$E[\Pi(Q^*_{\alpha(e)}, e)] = (p - v)\mu_x - C_\alpha(Q^*_{\alpha(e)}, e) - c(e)$$

$$= (p - v)\mu_x - \alpha(e) C(Q^*) - c(e)$$

Differentiating $E[\Pi(Q^*_{\alpha(e)}, e)]$ with respect to $e$, we get

$$\frac{\partial E[\Pi(Q^*_{\alpha(e)}, e)]}{\partial e} = -\frac{\partial \alpha(e)}{\partial e} C(Q^*) - \frac{\partial c(e)}{\partial e}$$

$$\frac{\partial^2 E[\Pi(Q^*_{\alpha(e)}, e)]}{\partial e^2} = -\frac{\partial^2 \alpha(e)}{\partial e^2} C(Q^*) - \frac{\partial^2 c(e)}{\partial e^2}$$
Once $\frac{\partial^2 E[\Pi(Q^*(e),e)]}{\partial e^2} < 0$, we can derive the optimal marketing effort $e^*$ by setting the partial derivative $\frac{\partial E[\Pi(Q^*(e),e)]}{\partial e}$ equal to zero, which in turn determines the optimal order quantity of the seller. Clearly, the second-order condition holds for a concavely decreasing $\alpha(e)$ and a convexly increasing $c(e)$. In this case, $e^*$ can be characterized by the following equation:

$$\frac{\partial \alpha(e)}{\partial e} C(Q^*) + \frac{\partial c(e)}{\partial e} = 0.$$  \hspace{1cm} (17)

where $C(Q^*)$ is the minimum expected cost when no marketing effort is devoted.

### 2.3 Demand and Stocking Quantity

Marketing researchers often argue that quantity displays can have a positive impact on demand and thus, ultimate sales. In essence two types of stimuli that impact demand positively due to the quantity displayed are: (a) a selective effect; and (b) an advertising effect. There is a significant body of literature on inventory systems where demand is positively impacted by stock level. These stock dependent inventory models developed so far can be classified into two categories. Models in the first category dealt with demand that is a function of the initial stock level, while the others assumed that demand is dependent on the instantaneous stock level and may thus vary over time. We will focus on newsvendor models with stock-dependent demand. Readers who are also interested in other inventory systems may refer to Urban [50] for a comprehensive review of the stock-level-dependent periodic-review models.

Baker and Urban [7] studied a deterministic single-period model in which demand at time $t$, $x(t)$, is a polynomial function of the instantaneous stock level, $i(t)$, and solved for the optimal order quantity $(Q)$, which is also the initial stock level, under the objective of maximizing the total profit. Specifically, they assumed a demand function $x(t) = ai^{1-b}(t)$, where $a > 0, 1 > b > 0$. The constant parameter $b$ is constrained to be some value between zero and one to ensure that the marginal increase in demand decreases as the stock level increases. Sensitivity analysis conducted in this paper showed that failing to consider the stimulating effect of stock on demand when it exists may lead to miserable optimization results, since the model is quite sensitive to changes in $b$. Urban and Baker [51] investigated a similar deterministic newsvendor problem with stock-dependent, price sensitive and time proportional demand. They expressed the demand rate as $x(p, t, i) = ap^{-k}t^{r-1}i^{1-b}$, where
Although no structural results were obtained for the general case, sensitivity analysis performed regarding the effects of price, time and stock level on demand reassured the importance of recognizing the correct demand pattern.

Urban [49] also examined a stock-dependent newsvendor model. Demand in this paper takes the same polynomial function as in Baker and Urban [7], but with the total stock level \( i(t) \) replaced by the observed stock level \( \varphi(t) \) assuming that customers are only influenced by the displayed products. That is, \( x(\varphi(t)) = a\varphi^{1-b}(t), a > 0, 1 > b > 0 \), where \( \varphi(t) = \min(s, i(t)) \) and \( s \) denotes the shelf space allocated to that product. With the further assumption of the quasi-concavity of the profit function, Urban derived the closed form expressions for the optimal order quantity \( Q^* \), which is equal to \( i^*(0) \), and the optimal shelf space \( s^* \) and investigated the interdependency between the two decisions. The relationship between \( Q^* \) and \( s^* \) can be characterized by the equation \( s^*/Q^* = (1-b)(p-c)/(1-b(p-c)+v) \), where \( v \) is the unit shelf-space cost. Urban also pointed out that the holding cost, which can be considered as the negative salvage cost in our setting, has an equal impact on the optimal values of the two decisions variables.

Most of the inventory models where demand is assumed to be a function of instantaneous stock level seldom incorporate the uncertainty of demand due to technical complexity. Similarly, when stochastic stock-dependent demand is considered in an inventory system, researchers usually interpret the word ”stock” as ”initial stock”, which can be viewed as ”order quantity” in a single period newsvendor model. In the rest of the section, some simple results will first be derived for a special case of the stochastic newsvendor model that uses initial stock level as independent variable, followed by a brief literature review of stochastic stock-dependent newsvendor models.

Assume that customer demand is linearly increasing in buyer stocks as follows\(^3\): \( x(Q) = a + bQ + \epsilon, a > 0, 1 > b > 0 \), where \( \epsilon \) is a random variable with density function \( h(\cdot) \) and distribution function \( H(\cdot) \)\(^4\). Assuming that the seller is risk neutral and would like to maximize the expected profit, the objective function for this setting is as follows:

\[
E[\Pi(p, Q)] = (p - v)(a + bQ + \mu_\epsilon) - c_o \cdot E[Q - x]^+ - c_u \cdot E[x - Q]^+ 
\]

\(^3\)The assumption of linearity is made for tractability but the results hold as long as demand is non-decreasing in quantity.

\(^4\)The mean of the random term \( \epsilon \) is not required to be zero for the linear additive demand form.
where

\[ E[Q - x]^+ = \int_0^{(1-b)Q-a} H(\epsilon) d\epsilon \]
\[ E[x - Q]^+ = \int_{(1-b)Q-a}^{+\infty} [1 - H(\epsilon)] d\epsilon \]

It is quite straightforward to show that the expected profit function is concave in \( Q \). By equating the first order derivative of the objective function with respect to \( Q \) to zero, we obtain

\[ H((1-b)Q^* - a) = \frac{p-u}{1-b} + B \]

\[ \frac{p}{p-g+B} \] (19)

where \( H((1-b)Q^* - a) = F(Q^*) \). As \( 1 > b > 0 \), the fractile value given by \( \frac{p-u}{1-b} + B \) is greater than the critical fractile value \( \frac{p-v}{p-g+B} \) for the classic newsvendor model. Hence, we can conclude that the stock-dependent demand in an additive form causes an increase in the order quantity as compared to the classic newsvendor model.

Dana and Petruzzi [17] investigated an extension of the classic newsvendor model where each customer is interested in maximizing her own expected utility and chooses whether to visit the firm or go for an outside option, \( u \), based on the firm’s price and initial stock level she observes as well as her own valuations of the product and the outside option. Given these assumptions of consumer behavior, the firm faces uncertain aggregate demand which can be modeled as a multiplicative function of both its price and its initial stock level. The authors considered two cases, the exogenous price case for a firm that is merely a price taker and the endogenous price case where price can be chosen optimally. Both the benchmark policy, which ignores the effect of stock on demand, and the optimal policy are determined in each case. The uniqueness of the optimal policies again requires specific restrictions on the distribution of consumers’ outside options and/or the distribution of aggregate demand as mentioned in the previous section. The comparison between the optimal policy and its corresponding benchmark policy showed that a firm that internalizes the effect of stock on demand usually holds more stock, provides higher service level and earns higher expected profit no matter it is a price taker or price setter. It is also worth mentioning that sequential optimization of price and stocking quantity can be applied in the endogenous price case.

Balakrishnan et al. [8] generalized the newsvendor model to incorporate the stochastic and initial-stock-level dependent demand. The authors proposed a general stochastic demand
model via an inverse fractile function to capture the stimulating effect of stock on demand. Let \( f(Q, x) \) denote the pdf of \( x(Q) \) and \( l(Q) \) denote the stock-dependent lower support of the pdf. The corresponding inverse fractile function is defined as \( \phi(Q, \rho) = F^{-1}(Q, \rho) \), where \( \rho \) is some given fractile value between 0 and 1. To ensure that demand increases at a decreasing rate with the initial stock level, \( \phi(Q, \rho) \) is assumed to be a concavely increasing function of \( Q \), which is later shown to be the sufficient condition for the concavity of the profit function. Applying a variable transformation \( x = \phi(Q, \rho) \), the expected sales for a given order quantity \( Q \) can be expressed as \( S(Q) = \int_0^{F(Q,Q)} \phi(Q, \rho) d\rho + Q(1 - F(Q, Q)) \). The marginal expected sales \( S'(Q) \) was found to consist of two parts, \( 1 - F(Q, Q) \) and \( \int_0^{F(Q,Q)} \phi'(Q, \rho) d\rho \). The former was interpreted as the availability effect of stocking higher quantities and the latter as the promotional effect of stocking more. When the stock stimulating effect is ignored, the term representing the promotional effect drops out in the optimality condition. By observing so, the authors extended, from a multiplicative demand form to a general demand form, the conclusion reached in Dana and Petruzzi (2001) that the consideration of the demand-stimulating effect of stock leads to higher initial stock levels and higher fill rates.

3 Supplier Cost and Stocking Quantity

Quantity discounts are a widely adopted pricing scheme in practice. Basically, the notion is that the larger the order quantity, the lower the unit procurement cost and the justification for its popularity includes, but not limited to, costs transfer, price discrimination and channel coordination. When a supplier has a high fixed cost, s/he may choose to offer quantity discounts to encourage larger orders, which helps achieve economies of scale. Considering that larger orders can also shift part of the overstock risk to the buyers, the suppliers are often more than willing to practice quantity discounts when inventory costs are high. As for price discrimination, quantity discounts helps suppliers to identify price-insensitive retailers from price-sensitive ones and charge higher prices (if possible) to retailers who are willing to pay more. The channel coordination explanation for quantity discount may not be so straightforward at first glance. We will discuss in detail as follows.

Consider a supply chain with a single supplier and a single newsvendor-type retailer facing uncertain demand. Assume that the two channel members have full information about demand and each other’s cost structure and that the retailer can only order before
the demand realizes. Then, the retailer’s and suppliers’ profits can be expressed as

\[ E[\Pi_R] = (p - v)\mu_x - (p - v + B)E[x - Q]^+ - (v - g)E[Q - x]^+ \]  \hspace{1cm} (20)

and

\[ E[\Pi_S] = (v - c)Q \]  \hspace{1cm} (21)

respectively, where \( c \) denotes the variable cost for the supplier. Since the supplier will not operate with negative or zero profit, it is reasonable to assume \( v > c \). The channel profit, which is the sum of the retailer’s and supplier’s profits, is

\[ E[\Pi_{SC}] = (p - c)\mu_x - (p - c + B)E[x - Q]^+ - (c - g)E[Q - x]^+ \]  \hspace{1cm} (22)

When the channel members are only interested in maximizing their own profits, the retailer will place an order of \( F^{-1}\left(\frac{p-v+B}{p-g+B}\right) \) units, which is smaller than the order quantity \( F^{-1}(\frac{p-c+B}{p-g+B}) \) that maximizes the channel profit. Hence, offering quantity discounts to retailer, which results in a lower wholesale price, can be a feasible solution to achieving channel coordination. However, it must be made clear that not all forms of quantity discounts can help obtain channel coordination.

According to Jeuland and Shugan [25], among various quantity discounts, only those that make channel members’ profit maximizing incentives consistent with overall channel profit maximization can coordinate the supply chain. This implies that channel members’ profit functions should be some fixed linear function of the channel profit under the channel-coordinating quantity discount. One possible form of this kind of quantity discounts was proposed in their work for a two-member supply chain under deterministic and price-dependent demand. That is, \( v(x) = k_1(p(x) - c - C) + (k_2/x) + c \), where \( k_1 \) and \( k_2 \) represents the fraction of channel profit and some fixed amount of profit allocated to the supplier, \( p(x) \) is the inverse of \( x(p) \), and \( C \) (\( c \)) is the variable cost on the retailer’s (supplier’s) part.

Jucker and Rosenblatt [26] studied the implications of quantity discounts for both the retailer and supplier in the context of the newsvendor problem. Consider a profit maximizing retailer that faces deterministic demand and an all-units quantity discount offered by the supplier. The authors showed that the retailer orders up to \( q_i \) if the amount of units she must purchase, denoted by \( Q' \), falls into the interval \( [b_i, q_i) \), and orders exactly \( Q' \) units if \( Q' \)
is in \([q_{i-1}, b_i]\), where \(b_i = q_{i-1} + g\) and \(g < v_n\). \(b_i\) can be interpreted as the breakeven point at which the retailer is indifferent between ordering \(b_i\) units and ordering \(q_i\) units. Since this kind of behavior reflects zero marginal cost of units in \([b_i, q_i]\), the all-units quantity discount can be characterized by a marginal cost function, \(\frac{\partial G(Q)}{\partial Q}\), that equals \(v_i\) for \(q_{i-1} \leq Q < b_i\) and 0 for \(b_i \leq Q < q_i\). Incremental quantity discounts can be modeled as a special case where \(b_i = q_i\). Using marginal analysis, the authors then proposed a new solution procedure for newsvendor problems with all-units quantity discount. Traditionally, the global maximum is found by solving the equation \(\frac{\partial G(Q)}{\partial Q} = v_i\) repeatedly from the highest unit cost \(v_0\) until the first feasible solution and comparing the resulting costs at that quantity and the larger price break quantities. However, the authors suggested that, for cases in which \(\frac{\partial G(Q)}{\partial Q} = v_i\) cannot be solved analytically, it is better to first compare the values of the marginal cost function and marginal revenue function from the lowest price break quantity to locate the intersections if exist. While price break quantities can be candidates for the global optimum in the case of all-units quantity discount, the paper mentioned that we only need to pay attention to the intersections when an incremental quantity discount is considered.

Pantumsinchai and Knowles [37] presented solution algorithms to the standard container size discounts in the newsvendor environment. The newsvendor determines the optimal order quantity and the number of different standard containers to make up the order under random demand. Suppose that there are \(m\) standard container sizes available with \(q_1 > q_2 > \ldots > q_m\). The unit purchase costs associated with container sizes satisfy the inequality \(0 < p_1 \leq p_2 \leq \ldots \leq p_m\). Let \(x\) denote the newsvendor’s initial inventory level and \(y_i\) denote the optimal order-up-to level that minimizes \(G(y_i, x)\), the purchase cost if applicable plus the expected overage and underage cost when only the \(i\)th container size is available. Define \(n_i = (y_i - x)/q_i, i = 1, 2, \ldots, m\). The authors showed that the newsvendor should not order when \(x \geq y_1\). In the case where \(x < y_1\), the optimal order quantity \(Q^*\) is bounded by \(n_uq_u\) and \((n_u + 1)q_u\), where \(n_u = (y_u - x)/q_u\) and \(u\) is chosen such that \(n_i = 0, i = 1, \ldots, u - 1\) and \(G_1((n_1 + 1)q_1 + x, x) \geq G_2((n_2 + 1)q_2 + x, x) \geq \ldots \geq G_u((n_u + 1)q_u + x, x)\). Solution algorithms are then proposed for solving the general case and the restricted case where the newsvendor is required to use successively smaller container sizes. In addition, when a fixed cost \(K\) is incurred for placing an order, the authors suggested comparing \(G(y^*, x) + K\) with \(G(x, x)\) and ordering if \(G(x, x)\) is larger.

Lin and Kroll [35] considered both all-units quantity discount and incremental discount
in the newsvendor problem with dual performance measures: \( \max \)imize the expected profit subject to a constraint that the probability of achieving a target profit level is no less than a predetermined risk level”. The retailer’s ordering decision was analyzed under four models: Model 1: all-units discount with zero shortage cost; Model 2: incremental discount with zero shortage cost; Model 3: all-units discount with positive shortage cost; and Model 4: incremental discount with positive shortage cost. For Model 1, using results from Lau [34], the authors showed that the constraint \( \text{Prob}(\Pi \geq T) \geq \theta \), where \( T \) is the target profit level and \( \theta \) is the predetermined risk level, actually imposes another feasible condition on \( Q \) associated with \( v_i \), i.e., \( T/(p-v_i) \leq Q \leq ((p-g)F^{-1}(1-\theta)-T)/(v_i-g) \). Therefore, the feasible interval for \( Q \) corresponding to \( v_i \) can be obtained by combining this interval with the interval \([q_{i-1}, q_i]\) in this case. The optimal order quantity can be found by solving the newsvendor model with an all-units quantity discount scheme. For Model 2, the decision variable \( Q \) has the same feasible interval as for Model 1 for each \( v_i \) and closed-form expression for the optimal solution can be derived. For the other two models with positive shortage cost, again using the results from Lau [34], the constraint on the risk level can be replaced by the inequality \( F(D_{2i} - F(D_{1i})) \geq \theta \), where \( D_{2i} = [T + (v_i-g)Q_i]/(p-g) \) and \( D_{1i} = [(p-v_i+B)Q_i - T]/B \). Since the constraint is nonlinear, no closed form solution can be obtained in these two cases.

Weng [53] studied the coordination issues of a single-manufacturer-single-distributor system under price-dependent stochastic demand. A two-part tariff policy, a fixed payment plus the wholesale price, was shown to be sufficient to achieve the coordination. The author assumed that all demand must be satisfied and a second order is allowed at a higher cost when demand exceeds the initial order quantity. Decisions to be made include the distributor’s selling price and initial order quantity as well as the manufacturer’s unit profit margin. The manufacturer’s selling price is equal to the sum of the unit profit margin and the unit production cost. In the absence of coordination, it is assumed that the manufacturer may not know the distributor’s actual response to its selling price and thus determines the unit profit margin based on its estimated order quantity, which may deviate from the distributor’s actual order quantity. When coordination is enabled, the manufacturer has complete information about the distributor’s response. The price-dependent random demand is modeled by a generalized phase-type distribution, i.e., \( F(x|p) = 1 - e^{-x/\mu(p)} \sum_{n=1}^{+\infty} a_n \sum_{i=0}^{n-1} \frac{(x/\mu(p))^i}{i!} \), where \( a_n, n = 1, 2, \ldots \) is some appropriately-chosen discrete probability distribution and
\( \mu(p) = dp^{-k}, \ d > 0, \ k > 1. \) The parameter \( k \) represents the sensitivity of demand to price. The optimal pricing and ordering policies with and without coordination are then characterized and contrasted under previous assumptions. It is shown analytically that the optimal initial order quantity with coordination is always greater than the optimal initial order quantity without coordination and the gap increases in \( k \). Two effects of coordination are found to contribute to the above result, the reduction in the unit overage and underage costs and the reduction in the manufacturer’s selling price. Numerical analysis demonstrated the necessity of coordination for both parties when demand is highly sensitive to the distributor’s selling price. The author also developed a two-part tariff policy to coordinate the system in which the manufacturer sets its selling price equal to its unit production cost and the distributor makes a fixed payment to the manufacturer.

In Weng [54], a generalized newsvendor model was developed to examine the effect of coordination for a single-manufacturer-single-buyer supply chain under stochastic demand and an all-units quantity discount was designed to coordinate the system. The only decision included in this model is the buyer’s order size. No specific distribution is assumed for the demand. Unfilled demand may be backordered at a cost and a second order may be placed with the manufacturer depending on whether the additional order generates positive profits for both parities. Let \( b \) denote the unit backorder cost, \( t_2 \) denote the ordering cost for the second order and \( s_2 \) be the manufacturer’s setup cost for the second run. The buyer is willing to place a second order if the unfilled demand, \( q \), satisfies the inequality
\[
(p - v - b)q - (t_2 + s_2) + Bq \geq 0,
\]
where the sum of the first two terms is the profit to be obtained from placing a second order and the last term is the shortage cost incurred if not placed. This modeling of unfilled demand can be used to cover situations in which all demand must be satisfied by setting \( B \to +\infty \) or no re-supply is available from the manufacturer by letting \( b = p - v + B \). Analytically, the author showed that, for an increasing concave demand distribution function, the system optimal order quantity, \( Q_{SC}^* \), is always larger than the buyer’s optimal order quantity, \( Q_B^* \) when coordination is not implemented. Let \( c_{\min}(Q_{SC}, \rho) \) denote the manufacturer’s lowest acceptable \( v \) satisfying \( E[\Pi_M(Q_{SC}^*)] \geq (1 + \rho)E[\Pi_M(Q_B^*)], \rho \geq 0 \). And let \( c_{\max}(Q_B^*, \gamma) \) denote the buyer’s highest acceptable \( v \) satisfying \( E[\Pi_B(Q_{SC}^*)] \geq (1 + \gamma)E[\Pi_B(Q_B^*)], \gamma \geq 0 \). If \( c_{\max}(Q_B^*, \gamma) \geq c_{\min}(Q_{SC}^*, \rho) \), the author argued that an all-units quantity discount in which the single price break quantity and the corresponding unit price is set to \( Q_{SC}^* \) and \( c_{\max}(Q_B^*, \gamma) \), respectively, can coordinate
the system since both parties are no worse off when coordinate. The two parameters \( \rho \) and \( \gamma \) specify the allocation of the increased system profit due to coordination. For example, if \( \rho = 0 \), then all increased system profit goes to the buyer, and vice versa.

Cachon and Lariviere [13] focused on the use of revenue-sharing contracts in supply chain coordination and offered comparisons between revenue-sharing and other contracts including quantity discounts. Again, consider a single-supplier-single-retailer supply chain. The profit-maximizing retailer makes two decisions, order quantity and retail price (shortage cost are not included). Let \( S(Q,p) \) be the expected sales for the quantity-price pair \((Q,p)\), \( c_s \) be the unit production cost of the supplier and \( c_r \) be the retailer’s unit operating cost. The retailer’s expected revenue can then be expressed as \( R(Q,p) + gQ \), where \( R(Q,p) = (p-g)S(Q,p) \), and the expected system profit \( E[\Pi_{SC}(Q,p)] = R(Q,p) - (c_s + c_r - g)Q \). Use \((Q^*,p^*)\) denote denote the quantity-price pair that maximizes the expected system profit. The authors demonstrated that, for a fixed retail price \( p^* \), the quantity discount scheme \( v(Q) = (1-\alpha)\frac{R(Q,p^*)}{Q} + \alpha(c_s + c_r) + (1-\alpha)g - c_r \) can also coordinate the supply chain. However, this specific quantity discount may fail to coordinate the system when the retail price deviates from \( p^* \), while the linear relationship between the retailer’s expected profit and the system’s expected profit maintains under revenue sharing. Furthermore, as \( R(Q,p) \) is included in \( v(Q) \), quantity discounts may only apply to a multi-retailer situation where the retailers have the same revenue function.

Lau, Lau and Wang [33] investigates the design of a supplier’s wholesale pricing policy in the form of an all-units discount with a single price-break point. They considered a supply chain with a single stackelberg-leader supplier and numerous heterogeneous newsvendor-type retailers. Each retailer \( i \) faces a normally distributed demand with mean \( \mu_i \) and standard deviation \( \sigma_i \), where \( \sigma_i = k\mu_i \) and all \( \mu_i \)s follow another stochastic distribution \( H(\cdot) \) by assumption. According to the author, all demand characteristics are to be estimated from empirical data and they assume that the parameters \( k, p \) and \( c \) are identical for all retailers. Based on a numerical analysis, they conclude that for a Stackelberg-leader supplier whose only objective is profit maximization, the optimal all-units quantity discount dominates the optimal single wholesale price scheme and that retailers’ heterogeneity, indicated by the standard deviation of \( H(\cdot) \), adversely affects the effectiveness of the quantity discount scheme. In addition, for a profit-maximizing supplier that is also concerned with allocating a certain amount of profits to the retailers, the optimal quantity discount again performs
better than the optimal single wholesale price scheme in terms of the supplier’s expected profit.

Burnetas, Gilbert and Smith [12] addressed a pricing-policy design problem of a profit-maximizing supplier that has less demand information than the newsvendor-type buyer does. The optimal all-units discount and the optimal incremental discount were also contrasted in terms of the effectiveness of increasing the supplier’s expected profit in this paper. The authors modeled information asymmetry by considering \( n \) buyer types that have different demand distribution \( F_i(x) \), \( i = 1, 2, \ldots, n \), but the same cost structure. For the supplier, each type \( i \) has a probability of \( p_i \) to be the real type of the buyer. And it is assumed that \( F_j(x) \) stochastically dominates \( F_i(x) \) for \( j > i \). The buyer has an outside supply source to get the products at a price of \( u, c < u < p \). In such an environment, following the revelation principle, the pricing mechanism design problem can be formulated as maximizing the supplier’s expected profit subject to constraints that each buyer type prefers the quantity-price option intended for him and accepting the intended option should yield at least as much expected profit as resorting to the outside supply source. The solution to this problem is referred to as the fixed-package pricing policy. The optimal all-units discount and the optimal incremental discount can be obtained by solving a similar problem. The authors were able to show that, both analytically and numerically, the fixed-package pricing policy dominates the optimal all-units discount, which in term outperforms the optimal incremental discount, when expected profit is concerned.

Ingene and Parry [23] showed the existence of a two-part tariff wholesale pricing policy that coordinates a supply chain with a single supplier and multiple independent retailers. It is assumed that the deterministic demand faced by each retailer is unique and concavely decreasing in retail price. The number of retailers in the supply chain is the number of retailers that obtain non-negative profits if participate and can be considered as a decision in this problem. Let \( b(Q_i) \) denote the unit payment by retailer \( i \) for an order of size \( Q_i \) and \( c \) denote the supplier’s marginal cost. Utilizing marginal analysis techniques, the authors showed that a two-part tariff in which \( b(Q_i) = c + \phi_c/Q_i \) coordinate the supply chain, where \( \phi_c \) is a fixed fee paid by each participating retailer to the supplier. The number of participating retailers under the coordination wholesale pricing policy, denoted by \( N_c^* \), should be no more than the number of participating retailers in the integrated system since each retailer in the decentralized system has a non-negative \( \phi_c \) to cover. Finally, it is shown that the coordinating
two-part tariff policy is different from the two-part tariff policy that maximizes the supplier’s profit except in the following two cases: the supply chain consists of a single retailer; or the sales of the retailer that achieves exactly zero profit from participation is equal to the total sales divided by the number of participating retailers.

Altintas, Erhun and Tayur [3] addressed the problem of designing the optimal all-units quantity discount with a single price break from the perspective of a supplier that faces a newsvendor-type buyer in a multiple-period setting. It is assumed that, for every period in the planning horizon except the last one, leftover units at the end of one period will be carried over to the next period and unfilled demand will be backlogged at a cost. At the end of the planning horizon, leftover units must be disposed and unfilled demand must be satisfied from an outside supply source. The supplier is concerned with maximizing his own profit, which is the amount of money coming from the retailer less the trucking cost. The trucking cost for each order of size $Q$ can be expressed as $K \lceil \frac{Q}{C} \rceil$, where $K$ is the fixed operating cost per truck and $C$ is the capacity of each truck. The buyer seeks to minimize her average cost. The authors first considered a single period model that allows positive initial inventory level $y$. Let $p_i$ denote the critical fractile value corresponding to the whole sale price $v_i$, $i = 0, 1$. Define $S_i = F^{-1}(p_i)$, $i = 0, 1$, and $S_{01}$ as some value that equates the resulting expected costs of ordering $q_1$ or ordering up to $S_0$ if necessary. The authors demonstrated that a three-index policy with indices $(S_0, S_1, S_{01})$ can be the buyer’s optimal ordering policy. To be specific, order $\max S_1 - y, q_1$ if the initial inventory level $y$ is less than $S_{01}$ and order $\max S_0 - y, 0$ otherwise. However, this three-index policy does not apply to a multi-period model according to the numerical study. One interesting finding is that demand variance and the variance of orders not including the orders of size 0 may move in opposite directions, since the buyer may follow polices with fewer indices under more variant demand. For the supplier’s problem, fixing $v_1$ and treating $v_0$ and $q_1$ as variables, the authors were able to show numerically that the supplier should offer quantity discounts that can act as minimum order quantity schemes to reduce the order variance when the fixed cost $K$ is high.

Lariviere and Porteus [31] studied the pricing policy of a Stackelberg-leader manufacturer that sells a product to a news-vending retailer. Their analysis focused on the impact of demand distribution characteristics on supply-chain performance, such as system efficiency and division of system profit between the partners. Anticipating the retailer’s ordering
decision, the manufacturer determines her wholesale price to maximize her own profit while assuring the retailer’s acceptance of this price. The retailer accepts the offer if the resulting expected profit exceeds her opportunity cost, which is set to zero when the manufacturer acts as a Stackelberg leader. To ensure the uni-modality of the manufacturer’s expected profit function, a condition was imposed on the demand distribution that it must have an increasing generalized failure rate (IGFR), a finite mean and an interval support. Let $x_i$ be the random variable representing the demand in market $i$, $i = 1, 2$. The authors showed that the manufacturer offers a higher wholesale price if $x_2$ is smaller than $x_1$ in the star order, which implies a lower coefficient of variation in market 2. Moreover, sufficiently reduced relative variability always improves system efficiency despite of the increased stock-out probability. The authors also pointed out that the optimal wholesale price they derived may only serve as an upper bound on the wholesale price chosen in practice due to factors such as the manufacturer’s incentive to include retailer in her demand forecasting process and the possibility of a powerful retailer instead of a powerful manufacturer as assumed in this paper.

We now provide the key results for different quantity discount schemes which could be used by the supplier assuming a setting in which the retailer operates as a newsvendor. The three types of quantity discount schemes we describe are: (a) linear quantity discount; (b) incremental-units quantity discount; and (c) all-units quantity discount. A linear discount scheme uses a wholesale price $v(Q) = v_0 - tQ$ where $t$ is some constant discount. All-units discount has different wholesale prices for different quantity intervals: $v_0$ for $0 \leq Q < q_1$, $v_1$ for $q_1 \leq Q < q_2$, $v_2$ for $q_2 \leq Q < q_3$, etc. These wholesale prices would apply to the entire order quantity $Q$. Incremental discount offers different wholesale prices for the incremental amount in different quantity intervals: $v_0 = w_0$ for $0 \leq Q < q_1$, $v_1 = (w_0q_1 + w_1(Q - q_1))/Q$ for $q_1 \leq Q < q_2$, $v_2 = (w_0q_1 + w_1(q_2 - q_1) + w_2(Q - q_2))/Q$ for $q_2 \leq Q < q_3$, etc. In all cases, we have $v_0 > v_1 > v_2 > \ldots$.

### 3.1 Linear Quantity Discount

Assuming a risk-neutral retailer facing with a linear quantity discount, the retailer’s expected profit function can be attained by simply replacing $v$ with $v(Q) = v_0 - tQ$ in its expected
profit function in the basic case.

\[ E[\Pi_R] = (p - v(Q))\mu_x - (p - v(Q) + B)E[x - Q]^+ - (v(Q) - g)E[Q - x]^+ \]  

By imposing certain parametric restrictions \(^5\), the optimal order quantity \(Q^*_i\) can be characterized by the equation

\[ F(Q^*_i) = \frac{p - v_0 + B}{p - g + B} + \frac{2tQ}{p - g + B}. \]  

Given that \(\frac{2tQ}{p - g + B} > 0\), it is obvious that the buyer would be induced to order more under linear discounting scheme as compared to the fixed price scheme. While a closed form expression for \(Q^*_i\) cannot be derived for an arbitrary demand distribution, the optimal value of \(Q^*_i\) can be found via a simple uni-dimension search algorithm.

### 3.2 All-units Quantity Discount

For simplicity, consider an all-units quantity discount with two price-break points as follows.

\[ v = \begin{cases} 
  v_0, & q_0 \leq Q < q_1; \\
  v_1, & q_1 \leq Q < q_2; \\
  v_2, & q_2 \leq Q.
\end{cases} \]

where \(q_0 = 0\). The expected profit function of the retailer under this pricing scheme is then

\[ E[\pi_R] = \begin{cases} 
  (p - v_0)\mu_x - (p - v_0 + B)E[x - Q]^+ - (v_0 - g)E[Q - x]^+, & 0 \leq Q < q_1; \\
  (p - v_1)\mu_x - (p - v_1 + B)E[x - Q]^+ - (v_1 - g)E[Q - x]^+, & q_1 \leq Q < q_2; \\
  (p - v_2)\mu_x - (p - v_2 + B)E[x - Q]^+ - (v_2 - g)E[Q - x]^+, & q_2 \leq Q.
\end{cases} \]

Define \(Q_A(v_i) = F^{-1}(\frac{2tQ}{p - g + B})\). For each expected profit function associated with \(v_i, \ i = 0, 1, 2\), the optimal order quantity \(Q^*_A(v_i)\) is set to \(Q_A(v_i)\) if \(q_{i-1} \leq Q_A(v_i) < q_i, q_{i-1}\) if \(Q_A(v_i) < q_{i-1}\), and \(q_i\) if \(Q_A(v_i) > q_i\). The optimal order quantity of the retailer can be obtained by comparing the resulting expected profits \(E[\pi_R(Q^*_A(v_i))]\). For all-units quantity discounts with numerous price-break points, one possible procedure is to solve for the first \(Q^*_A(v_i)\) such that \(Q^*_A(v_i) = Q_A(v_i)\) for successively larger unit prices, compare the value of the corresponding expected profit function at that point with the values at all larger price break quantities, and choose the quantity that offers the largest expected profit.

\(^5\)These restrictions are: \(0 < t < \min[0.5f(Q)(p - g + B), (v - g)Q^{-1}]\) for all possible values of \(Q\).
3.3 Incremental Quantity Discount

Consider an incremental discount scheme with two price-break points. The wholesale price is set as follows

\[
\begin{align*}
v &= \begin{cases} 
v_0(Q) &= w_0, & 0 \leq Q < q_1; \\
v_1(Q) &= (w_0q_1 + w_1(Q - q_1))/Q, & q_1 \leq Q < q_2; \\
v_2(Q) &= (w_0q_1 + w_1(q_2 - q_1) + w_2(Q - q_2))/Q, & q_2 \leq Q.
\end{cases}
\end{align*}
\]

Then, the retailer’s expected profit function is

\[
E[\pi_R] = \begin{cases} (p - v_0(Q))\mu_x - (p - v_0(Q) + B)E[x - Q]^+ - (v_0(Q) - g)E[Q - x]^+, & 0 \leq Q < q_1; \\
(p - v_1(Q))\mu_x - (p - v_1(Q) + B)E[x - Q]^+ - (v_1(Q) - g)E[Q - x]^+, & q_1 \leq Q < q_2; \\
(p - v_2(Q))\mu_x - (p - v_2(Q) + B)E[x - Q]^+ - (v_2(Q) - g)E[Q - x]^+, & q_2 \leq Q.
\end{cases}
\]

Define \(Q_I(v_i) = F^{-1}(\frac{E[x + \mu_x]}{p - g + B})\). Again, for each expected profit function associated with \(v_i, i = 0, 1, 2\), set the optimal order quantity \(Q_I^*(v_i)\) to \(Q_I(v_i)\) if \(q_i - 1 \leq Q_I(v_i) < q_i\), \(q_i - 1\) if \(Q_I(v_i) < q_i - 1\), and \(q_i\) if \(Q_I(v_i) > q_i\). Different from all-units quantity discount, for incremental discount, we only need to search from the \(Q_I^*(v_i)\) that is equal to \(Q_I(v_i)\). The optimal order quantity is the one that yields the greatest expected profit.

4 Buyer Risk Profile

This part of the paper is mainly devoted to the analysis of the ordering policies of newsvendors with various preferences, including, but not limited to, risk-averse and risk-seeking preferences. Motivated by empirical and experimental observations, alternative risk preferences (such as loss-aversion, and stock-out aversion) have been analyzed in the context of the newsvendor setting. General utility functions, mean-variance, and coherent risk measures are the three most popular approaches to date. In what follows, we will introduce some general results for a risk-averse newsvendor derived and provide a detailed review of other contributions in this domain.

Eeckhoudt, Gollier and Schlesinger [18] provided a detailed investigation of the effects of risk, risk aversion and changes in various price and cost parameters for a risk-averse retailer. One importance assumption that simplifies the analysis is that all demand must be satisfied. To be specific, when the initial order is insufficient to fulfill the demand, the retailer purchases additional products at a cost of \(\hat{v}\), which is higher than the initial order.
cost \( v \). The profit function can then be formulated as

\[
\Pi(x, Q) = \begin{cases} 
\Pi_-(x, Q) := w_0 + px - vQ + g(Q - x), & x \leq Q; \\
\Pi_+(x, Q) := w_0 + px - vQ - \hat{v}(x - Q), & x > Q
\end{cases}
\]

where \( w_0 \) is the newsvendor’s initial wealth. Obviously, for a given \( Q \), profit increases in demand \( x \), which is defined on the interval \([0, T]\). For a risk-averse newsvendor, the objective is to maximize the expected utility of the random profit. Let \( u(\cdot) \) denote the increasing and concave utility function. The problem can be stated as \( \max E[u(\Pi(x, Q))] \), where

\[
E[u(\Pi(x, Q))] = \int_{Q}^{\infty} u(\Pi_-(x, Q)) f(x) dx + \int_{Q}^{\infty} u(\Pi_+(x, Q)) f(x) dx
\]

(25)

The first and second derivatives of the objective function with respect to \( Q \) are as follows.

\[
\frac{\partial E[u(\Pi(x, Q))]}{\partial Q} = (g - v) \int_{0}^{Q} u[\Pi_-(x, Q)] f(x) dx + (\hat{v} - v) \int_{Q}^{\infty} u[\Pi_+(x, Q)] f(x) dx
\]

\[
\frac{\partial^2 E[u(\Pi(x, Q))]}{\partial^2 Q} = (g - v)^2 \int_{0}^{Q} u'[\Pi_-(x, Q)] f(x) dx - (\hat{v} - g) u[pQ - vQ] f(Q) + (\hat{v} - v)^2 \int_{Q}^{\infty} u'[\Pi_+(x, Q)] f(x) dx
\]

It is relatively easy to show that the objective function \( E[u(\Pi(x, Q))] \) is concave in \( Q \) and thus the optimal order quantity can be determined by first-order condition. Let \( Q^* \) and \( Q^*_A \) denote the risk-neutral and risk-averse optimal order quantities, respectively. As

\[
u'[\Pi_-(x, Q)] \geq \nu'[pQ - vQ], \quad x \leq Q;
\]

\[
u'[\Pi_+(x, Q)] < \nu'[pQ - vQ], \quad x > Q,
\]

we get:

\[
\frac{\partial E[u(\Pi(x, Q^*))]}{\partial Q} = (g - v) \int_{0}^{Q_N} u'[\Pi_-(x, Q)] f(x) dx + (\hat{v} - v) \int_{Q}^{\infty} u'[\Pi_+(x, Q)] f(x) dx
\]

\[
< u'(pQ^* - vQ^*) \cdot [(g - v) \int_{0}^{Q_N} f(x) dx + (\hat{v} - v) \int_{Q}^{\infty} f(x) dx]
\]

\[
= 0
\]

This indicates that a risk-averse newsvendor orders less than a risk-neutral newsvendor in this setting. Keren and Pliskin [28] also show that, under uniformly distributed demand, the
optimal order quantity decreases in the degree of risk aversion even if penalty is incurred for lost sales. As an increase in risk aversion can be modeled by a concave transformation of the current utility function, it is not so difficult to show that \( Q^*_A \) decreases in the degree of risk aversion. Eeckhoudt, Gollier and Schlesinger [18] also showed that increases in the salvage value and the re-order cost both lead to a larger initial order, while the effect of changes in the initial order cost \( v \) on \( Q^*_A \) is still ambiguous. As for the changes in retail price, the authors showed that \( Q^*_A \) increases in \( p \) if the retailer’s preference displays decreasing partial relative risk aversion. The effects of increases in two types of risk were examined in this paper, risk associated with the initial wealth and demand risk. The additional background risk is introduced to the model by replacing \( w_0 \) with \( w_0 + \epsilon \), where \( \epsilon \) is a random variable with mean zero. It is shown that the optimal size of the initial order is unaffected by the additional risk if the utility is quadratic or exhibits constant absolute risk aversion, decreases if the utility displays decreasing absolute risk aversion. Riskier demand is described by a probability distribution \( G \) which is a mean-preserving with an increase in risk (MIR) of \( F \). Let \( Q^*_AF \) and \( Q^*_AG \) denote the optimal order quantity associated with demand distribution \( F \) and \( G \), respectively. The authors were able to prove that \( Q^*_AF > Q^*_AG \) if an MIR is restricted to \([Q^*_AF, T]\) or \( G \) and \( F \) satisfies the inequality \( [G(x) - F(x)](Q - x) \geq 0 \), \( Q^*_AF < Q^*_AG \) if MIR is restricted to \( [Q^*_AF, T] \).

Lau [34] provided solution procedures for solving the newsvendor problems under two alternative optimization objectives, maximizing expected utility and maximizing the probability of achieving a target profit level. Let \( A \) denote the actual sales of the newsvendor. The profit can be expressed as \( \Pi(x, Q) = (p - g + B)A - Bx - (v - g)Q \), where \( A = \min(x, Q) \). When only the tradeoff between mean and standard deviation of the profit is considered, the optimization problem becomes \( \max E[\Pi(x, Q)] - k \sqrt{\text{var}(\Pi(x, Q))} \), where \( k \) is some constant that reflects the decision maker’s degree of risk aversion. The author demonstrated that the optimal solution to the classic newsvendor problem always serves as an upper bound on the optimal order quantity in this scenario. Besides, he suggested determining the optimal solution value numerically due to the complexity of the first-order condition.

For the case in which the decision maker is concerned with maximizing the profit’s NW expected utility, the author considered a utility function approximated by an nth degree polynomial, \( u(\Pi(x, Q)) = \sum_{i=0}^{n} a_i \Pi^i(x, Q) \) and derived the corresponding first-order condition. Under the objective of maximizing the probability of achieving a target profit level \( T \),
i.e., \( \max \text{prob}(\Pi(x, Q) \geq T) \), the optimal order quantity is equal to \( T/(p - v) \) when \( B = 0 \), which is independent of demand distribution. When \( B > 0 \), the first order condition can be expressed as \( \frac{f(D_1)}{f(D_2)} = \frac{(p-v+B)(p-q)}{B(v-q)} \), where \( D_1 \) and \( D_2 \) are the two demand realizations that generates the target profit level \( T \). However, no conclusion can be reached regarding the behavior of the objective function for an arbitrary demand distribution.

Bouakiz and Sobel [9] showed the optimality of a base-stock policy for a multi-period newsvendor problem in which the newsvendor’s risk attitude can be described by an exponential utility function. Specifically, they assumed a utility function \( u(C(N)) = -e^{-\mu C(N)} \), where \( \mu \) is some positive constant measuring the risk-aversion degree of the retailer and \( C(N) \) is the present value of the total cost incurred in the \( N \)-period Planning horizon. Let \( \beta \) denote the discount factor. With the infinite-horizon model, the authors proved the existence of a functions \( y(\cdot) \) that gives the optimal base stock level for period \( i \) in the form of \( y(\beta^{i-1}\mu) \), which eventually approaches the optimal base stock level for a risk-neutral retailer.

Agrawal and Seshadri [1] considered a similar problem as Eekhoudt, Gollier and Schlesinger [18] except that the demand now depends on the retail price. Assuming that the mean demand is equal to \( a(p) \), which is a decreasing and concave function of \( p \), the authors modeled the effect of price on demand in two ways: (1) \( x = a(p)\epsilon_1 \), where \( \epsilon_1 \) is a random variable with unity mean; (2) \( x = a(p) + \epsilon_2 \), where \( \epsilon_2 \) is a random variable with mean zero. Through a graphical analysis of price \( p \) versus the value \( a(p)\xi \) for a given \( \xi \), the authors were able to reduce the original two-decision problem into a single-decision problem and then examine the impact of risk aversion on the retailer’s pricing and ordering decisions. With the multiplicative demand model (model 1), it is demonstrated that the optimal retail price increases in the degree of risk aversion, while the optimal size of the initial order decreases. With the additive demand model (model 2), a more risk-averse retailer will choose a lower retailer price, but the effect of risk aversion on the optimal initial order quantity cannot be precisely identified.

Schweitzer and Cachon [43] provided a complete investigation of the relationship between the newsvendor’s profit-maximizing order quantity and optimal order quantities under various alternative objectives. A symmetric demand distribution and zero shortage cost are assumed in the analysis. Let \( w_0 \) denote the initial wealth. For a loss-averse newsvendor, the authors considered a utility function in which \( u(w) \) is equal to \( w \) for \( w \geq w_0 \) and \( \lambda w \) otherwise, where \( \lambda \) is a measure of the newsvendor’s degree of loss aversion. It is proved that
the optimal order quantity of a loss-averse newsvendor is smaller than the profit-maximizing order quantity and decreases in $\lambda$. For a newsvendor with preference for waste-aversion (dislike of excess inventory), stock-out aversion, or minimizing ex-post inventory error (minimizing absolute differences between $Q$ and $x$), the authors modeled the respective utilities as expected profit less the cost incurred by the corresponding deviation. They show that a waste-averse newsvendor orders less a risk-neutral newsvendor, while a stockout-averse newsvendor orders more. By defining products with the critical fractile greater than 1/2 as high-profit products and as low-profit products otherwise, then to minimize ex-post inventory error, the newsvendor tends to order less than profit-maximizing quantity for high-profit products and more for low-profit products. The same ordering policy applies to newsvendors adopting an anchoring policy in which the decision maker anchors on mean demand and adjusts the profit-maximizing order quantity. The results derived from the last two situations are exactly what the authors observe in their experiments.

Arcelus, Kumar and Srinivasan [6] investigated the pricing, ordering and promotion policies of a risk-sensitive (risk-averse or risk-seeking) newsvendor under price-dependent and stochastic demand. Three promotion schemes are considered in this paper: rebate, advertising and the simultaneous use of the previous two schemes. It is assumed that the price and promotion decisions (i.e., rebate’s face value and advertising expenditure) only affect the deterministic part of demand. To be specific, price and rebate decisions has an exponential relationship with demand, while advertising enters in logarithmic form. Both additive and multiplicative demand errors are allowed in this model. It is also worth mentioning that, different from most of the previous papers we have reviewed, risk preferences are included in this model by introducing a risk-adjustment factor $\lambda$ for the stochastic part of the expected profit. $\lambda > 1$ for risk-averse newsvendors and $\lambda < 1$ for risk-seeking decision makers. Necessary optimality conditions are derived for all the possible decisions to be made for both additive error and multiplicative error. It is demonstrated that both the optimal order quantity and the optimal price are independent of the advertising expenditure. Since no closed form solution can be achieved due to technical complexity, numerical analysis is conducted for an example with uniformly distributed demand. One observation is that if no promotion scheme is available, price and expected profit increases in the value of $\lambda$, but order quantity falls.

Many researchers have considered the use of financial pricing and risk measures in in-
ventory problems. Anvari [4] argued that the mean-variance approach proposed in Lau [34] may not apply to a decision maker facing with multiple investment options since the effect of covariance cannot be captured by the mean-variance approach. The author hence suggested the use of the capital asset pricing model (CAPM) in the inventory models and thus optimizing from the perspective of the shareholders, instead of the managers, to avoid agency problem. Consider a newsvendor with two investment options, the inventory investment option (purchase $Q$ units at a price of $v$) and the option representing all the other investment opportunities other than the inventory option with a random return rate. It is assumed that $B = 0$ and that investment is made at the beginning of the single period while returns will be collected at the end of the period. Let $C$ denote the total capital available for investment. The objective is now to determine $Q$ maximizing the shareholder’s wealth represented by

$$[E[D(Q)] - \Omega \text{Cov}(D(Q), M)]/(1 + r_f) - C,$$

where $D(Q)$ is the random liquidating dividend, $\Omega$ is the market price per unit of risk, $M$ is the value of the market portfolio at the end of period and $r_f$ is the risk-free rate. The behavior of Cov$(D(Q), M)$ was examined in this paper under the assumption that the joint distribution of demand $x$ and $M$ is bivariate normal. (A solution procedure was developed in Chung [16] for this special case. ) Moreover, the author showed that the proposed model can be reduced to the classic newsvendor model by: (1) setting $r_f = 0$; (2) dropping out the term Cov$(D(Q), M)$ since risk is not considered in the classic setting; and (3) ignoring the return from the other investment option.

Gaur and Seshadri [20] investigated the problem of hedging operational risk using financial instruments in the newsvendor environment. Demand is assumed to be correlated with the price of a financial asset in a complete financial market with a unique risk-neutral pricing measure. No penalty is considered for lost sales. The decision maker makes the inventory decision at time 0 and sells the products at time $T$. Let $S_T$ denote the price of the financial asset at time $T$. Demand is modeled as $x = a + bS_T$ when demand is perfectly correlated with the asset price and as $x = a + bS_T + \epsilon$ when demand is only partially correlated. $\epsilon$ is the decision maker’s subjective forecast error. The authors were able to show that, for a risk-neutral decision maker, the optimal order quantity is not affected by the hedging decision in both cases. However, hedging transactions enable greater reduction in risk when demand is more correlated with the price of the asset. Under certain conditions on the structure of the hedging portfolio and the decision maker’s utility function, financial hedging increases the optimal order quantity if a risk-averse decision maker is willing to invest in inventory rather
than put all the money in the financial market.

Ahmed, Cakmak and Shapiro [2] suggested the use of coherent risk measures in inventory problems to obtain convex objective function. They analyzed a multi-period inventory problem with linear costs. The total cost for the single-period problem is expressed as
\[
C(Q) = vQ - \gamma g(Q - x) + B[x - Q]^+ + h[Q - x]^+,
\]
where \(\gamma\) is a discount parameter within the interval \((0, 1]\) and \(h\) is the holding cost for each unit left. Let \(\rho(\cdot)\) denote a coherent risk measure. When risk aversion is considered, the problem can be formulated as \(\min \rho(C(Q))\). For an arbitrary coherent risk measure, the authors proved its one-to-one correspondence with the min-max formulation. Using a mean-absolute deviation risk measure in the form of
\[
\rho(z) = E[z] + \lambda E[|z - E[z]|], \quad \lambda \in [0, 1/2],
\]
the structure of the optimal solution is found to be similar to that of the risk-neutral case for the multi-period problem.

Gotoh and Takano [22] investigated the risk-sensitive news-vendor problem under the objective of minimizing the conditional value-at-risk (CVaR), which is a coherent risk measure. This minimization problem is to determine \(Q\) and \(\alpha\) to minimize the term \(\alpha + \frac{1}{1-\alpha} \int_0^\infty [\Delta - \alpha]^+ dF(x)\), where the loss function \(\Delta\) can be defined as net loss \(-\Pi(Q)\) or total loss \(C(Q)\). \(\alpha\) is some pre-specified target level of the particular loss function. Using some of the results from Rockerfellar [41] and Rockafellar and Uryasev [42], the authors proved the convexity of the objective function and derived the closed form expressions for the two decisions \(Q\) and \(\alpha\) in both cases. Furthermore, it is showed that the \((Q, \alpha)\) solutions are not identical generally, which is different from what happens in the classic newsvendor case where maximizing profit is equivalent to minimizing cost. Analytical results are also obtained with the mean-risk criterion where variability is measured by the net loss CVaR. Linear programming formulation is then developed for a scenario where the decision maker has multiple products to handle.

Jammernegg and Kischka [24] analyzed the ordering policy and its corresponding performance measures, such as cycle service level and probability of loss, of a risk-sensitive news-vendor under the mean-deviation criterion. Again, no penalty cost is considered. The problem is formulated as
\[
\max_{(Q, \alpha)} \frac{\lambda}{1-\alpha} CVaR_\alpha(Q) + \frac{1-\lambda}{1-\alpha} E[\Pi(Q)],
\]
where \(CVaR_\alpha(Q) = E[\Pi(Q)|\Pi(Q) \leq F^{-1}(\alpha)]\), \(0 < \alpha \leq 1\), \(0 \leq \lambda \leq 1\). This objective applies to a risk-neutral newsvendor if \(\alpha = \lambda\), a risk-averse newsvendor if \(\alpha < \lambda\), and to a risk-seeking newsvendor if \(\alpha > \lambda\). The closed form expression for the optimal order quantity is also derived under this criterion. Specifically, \(Q^* = F^{-1}(\frac{\beta - v}{p - g} + \frac{\alpha - \lambda}{1-\lambda} \cdot \frac{v - g}{p - g})\) for \(\lambda \leq \frac{\beta - v}{p - g}\), and \(F^{-1}(\frac{\beta - v}{p - g} \cdot \frac{\alpha}{\lambda})\), otherwise. The conclusion is reached that a risk-averse (risk-seeking) newsvendor orders less (more) than a risk-neutral
newsvendor does. The cycle service level, probability of loss, defined as $P(\Pi(Q^*) \leq 0)$, and expected loss, defined as $E[\Pi(Q^*)]\Pi(Q^*) \leq 0]$, increase in $\alpha$ and decrease in $\lambda$. In addition to the conditional expected value approach discussed above, the authors also suggested using performance measures to model the decision maker’s risk preference. For example, a decision maker is risk averse (risk-seeing) if his/her self-specified CSL is less (greater) than the optimal CSL in the classic newsvendor scenario.

Wang and Webster [52] examined the ordering policy of a loss-averse newsvendor. They used the same utility function to describe the loss-aversion preference as in Schweitzer and Cachon (2000)[43] but assumed a positive shortage cost. Let $q_1(Q) = (v - g)Q$ and $q_2(Q) = \frac{p - v + B}{B}Q$. The objective function can be written as $E[u(\Pi(Q, x))] = E[\Pi(Q, x)] + (\lambda - 1)\left[\int_0^{q_1(Q)} \Pi_-(Q, x) dF(x) + \int_{q_2(Q)}^{\sup I} \Pi_+(Q, x) dF(x)\right]$, where $I$ is the feasible interval for demand $x$ and $\lambda \geq 1$ is the degree of loss aversion. The two integrals in the objective function represent the expected underage and overage losses. Define $(p - v + B)(1 - F(q_2(Q)))$ as the marginal underage loss and $(v - g)F(q_1(Q))$ as the marginal overage cost. The authors showed that, for $\lambda > 1$, whether a loss-averse newsvendor orders more or less than a risk-neutral newsvendor depends on the relative magnitude of the marginal underage and overage losses, which in terms depends on the demand distribution and the values of the price and cost parameters. If the marginal underage loss exceeds the marginal overage loss, the loss-averse newsvendor tends to order more than a risk-neutral decision maker. However, when the shortage cost is lower than some threshold value, the loss-averse optimal order quantity is always less than the profit-maximizing order quantity and decreases in the degree of loss-aversion. And it is not surprising to know that the loss-averse optimal order quantity increases in the shortage cost.

5 Selective Newsvendor

Selective newsvendor is referred to a newsvendor that faces multiple demand sources and has the flexibility to choose one or more demand sources available to serve. Kouvelis and Gutierrez [29] and Carr and Lovejoy [14] addressed newsvendor problems with several demand sources available but did not consider the selection problem. Sen and Zhang [44] analyzed the ordering and allocation policies of a newsvendor having several demand classes to serve and mentioned the possibility of closing certain markets to improve the expected
The generalized newsvendor problem studied in Kouvelis and Gutierrez [29] was motivated by the presence of geographically dispersed markets providing several selling opportunities of a product. The performance of the centralized control policy and that of a decentralized control policy are contrasted in this paper. The authors considered a cost-minimizing firm with a local production center in each of the two markets, demands of which are realized sequentially. The one with an early demand realization is referred to as the primary market, or market 1, and the other as the secondary market, or market 2. Transshipment of some of the leftover units from the primary market to the secondary market are allowed at a positive cost, which is transportation cost under the centralized control policy and the sum of transportation cost and some transfer price under the decentralized control policy. The currency exchange risk, modeled as a random exchange rate between the currencies in the two markets, is also included in the problem. The exchange rate is observed after the selling opportunity of the primary market but before the selling opportunity of the secondary market. Under the centralized control policy, decisions to be made by the firm include: local production quantity in market 1, amount of leftovers to be shipped from market 1 to market 2, and the local production quantity in market 2. The authors proved the concavity of the objective function and provided the optimal production and shipping policies. In addition, it is showed that the optimal production quantity for market 1 is non-decreasing with the presence of market 2. Compared to the centralized control policy in terms of the financial criterion, the decentralized control policy always results in sub-optimal production decisions no matter what transfer price is agreed upon. To capture the operational benefits of a decentralized system, the authors then suggested a decentralized system with intermediate purchasing coordination in which a third party purchases from market 1 and sells to market 2 to mimic the tradeoffs under the centralized control policy. Approximations of the nonlinear transfer pricing scheme is also provided in the paper.

Sen and Zhang [44] investigated the allocation rule and ordering policy of a profit-maximizing newsvendor that faces multiple independent demand classes with exogenous retail prices. Demands are assumed to realize sequentially over time. No salvage or shortage
costs are included in the modeling. Suppose there are $n$ demand classes. Let $p_i$ denote the retail price for demand class $i$. The authors considered two cases in this paper, the decreasing price case with $p_1 \geq p_2 \geq \cdots \geq p_n$ and $p_1 > v$ and the increasing price case with $p_1 \leq p_2 \leq \cdots \leq p_n$ and $p_n > v$. In the former situation, the optimal allocation rule is to satisfy the demand classes with higher retail prices first with no stock reservation for later demands. The necessary optimality condition was then derived, which can be interpreted as equating the expected marginal revenue to the marginal cost. The authors also proved that the optimal order quantity in this case is bounded by the optimal order quantities when $p_1$ or $p_n$ is applied to all demand classes, that is, $F^{-1}\left(\frac{v}{p_n}\right) \leq Q^* \leq F^{-1}\left(\frac{v}{p_1}\right)$, where $F(\cdot)$ is the distribution of the aggregate demand. For the increasing price case, a two-demand-class model was examined analytically. The authors adopted a static allocation scheme under which at least a certain amount of the quantity ordered can be reserved for the demand class with higher retail price. Let $P$ denote the pre-specified stock level that can be used to satisfy the lower-price demand and $s$ denote the portion of unsatisfied lower-price customers who are willing to purchase later at the higher price. A case-by-case analysis is then conducted to characterize the optimal order quantity and $P^*$. The authors suggested the decision makers close the lower-price market when the expected unit revenue to be obtained from the higher-price market, $sp_2$, exceeds the unit revenue from the lower-price market, $p_1$.

Carr and Lovejoy [14] analyzed a so-called inverse newsvendor problem in which a newsvendor with random capacity selects a demand distribution from a set of feasible distributions for each customer class to maximize the expected total profit. Demands and capacity are all assumed to be normally distributed with known means and variances. Customer classes are served sequentially according to a given priority list. Let $x_i$ denote the demand from customer class $i$ and $C_i$ denote the capacity available for customer class $i$. Suppose $C_1 (x_k)$ follow the normal distribution with mean $\mu_i$ and variance $\sigma_i^2$. The expected total profit can be expressed as $E[\Pi(\mu_1, \ldots, \mu_n, \sigma_1, \ldots, \sigma_n)] = \sum_{i=1}^{n}[a_i\mu_i + (a_i + b_i)(E[min(0, C_{i+1})] - E[min(0, C_i)])]$, where $C_i = C_1 - \sum_{j=1}^{i-1} x_j$, $a_i$ is the profit margin for each unit sold to customer class $i$ and $b_i$ is the penalty cost for each unfilled unit in customer class $i$. The expected profit function is showed to be concave in $(\mu_1, \ldots, \mu_n, \sigma_1, \ldots, \sigma_n)$. When there is only one customer class, the original optimization problem with two variables $(\mu_1, \sigma_1)$ can be reduced to a problem with a single variable $\mu_1$, i.e., $\max E[\Pi(\mu_1, \gamma(\mu_1))]$, where $\gamma(\mu_1)$ is a function that can be used to define the efficient
frontier of the feasible set. However, the new optimization problem may not be concave in \( \mu_1 \). If concavity holds for the new objective function and interior solutions exist, the optimal mean demand can be determined by the equation 
\[
\Phi(m) - \phi(m) \frac{\gamma(\mu_1^*)}{s} \gamma'(\mu_1^*) = \frac{b_1}{a_1 + b_1},
\]
where 
\[
s = \sqrt{\sigma_1^2 + \sigma^2}, \quad m = \frac{\mu - \mu_1^*}{s}.
\]
For the multi-class scenario, the authors proved that the optimal mean demand for each class is non-decreasing in the number of classes for increasing efficient frontier.

Taaffe, Geunes and Romeijn [47] focused on the market selection and ordering decisions in the context of the newsvendor problem. The market demands are assumed to be stochastically independent, marketing-effort dependent, and normally distributed with mean \( \mu_i \) and variance \( \sigma_i^2 \). Specifically, the mean demands and variances are both non-decreasing and bounded functions of marketing effort. Unfilled demand in each market will be satisfied later at a higher unit cost \( \hat{v} \). Use a binary vector \( y \) denote the market selection decisions, where \( y_i = 1 \) if the newsvendor chooses to enter market \( i \) and 0 otherwise. As the expected profit function is concave in the total order quantity \( Q_y \) for a given \( y \), the optimal total order quantity can then expressed as 
\[
Q^*_y = \sum_{i=1}^{n} \mu_i y_i + \Phi^{-1}(\rho) \sqrt{\sum_{i=1}^{n} \sigma_i^2 y_i},
\]
where \( \rho = \frac{\hat{v} - v}{v - g} \) and \( \Phi \) is the standard normal distribution. Let \( a_i \) denote the amount of marketing effort in market \( i \), \( t_i \) denote the unit cost of the marketing effort devoted to market \( i \), and \( S_i \) the fixed cost of entering market \( i \). Substituting the expression for \( Q^*_y \) into the original objective function and rearranging terms yields the so called selective newsvendor problem: 
\[
\max \sum_{i=1}^{n} (p_i - v) \mu_i(a_i) - ta_i - S_i) y_i - [(v - g) \Phi^{-1}(\rho) + (\hat{v} - v) L(\Phi^{-1}(\rho))] \sqrt{\sum_{i=1}^{n} \sigma_i^2(a_i) y_i}
\]
subject to the binary constraints on \( y_i \)'s, where \( L(\cdot) \) is the standard normal loss function.

The optimal marketing effort decisions are derived with further assumptions on \( \mu_i(a_i) \) and \( \sigma_i(a_i) \). Define \( \bar{p}_i = (p_i - v) \mu_i(a_i) - t_i - S_i \). The market selection solution can be found by utilizing the decreasing-expected-revenue-to-uncertainty-ratio \( (\frac{\bar{p}_i}{\sigma_i} \frac{a_i}{a_i}) \) property, that is, if an optimal selection solution includes market \( i \), it must include all markets with a higher or equal DERU ratio. However, the DERU property no longer holds when there are limited marketing resources available. In this case, a tailored branch-and-bound algorithm is developed in this paper to solve the nonlinear integer optimization problem.

Taaffe, Romeijn and Tirumalasetty [48] considered a similar selective newsvendor problem as in Taaffe, Geunes and Romeijn [47] except that the demands to be selected are all Bernoulli distributed and independent of marketing effort. Exact solution algorithms and heuristics are developed and evaluated for both the single-period and multi-period problems. Let \( p_i \) denote
the probability that demand $x_i$ materializes at $d_i > 0$ during the selling season. The single-period selective newsvendor problem with all-or-nothing demand can be formulated as: 

$$\max \sum_{i=1}^{n} ((p_i - g) E[x_i] - S_i)y_i - (v - g)Q - (\hat{v} - v) \sum_{\omega \in \Omega} P_\omega u_\omega \text{subject to } u_\omega \geq \sum_{i \in I_\omega} E[x_i]y_i - Q$$

and the non-negativity and binary constraints on the decision variables, where $P_\omega$ is the probability that a scenario $\omega$ realizes and $I_\omega$ is the set of demands that materializes in scenario $\omega$. A cutting-plane algorithm based on L-shaped method is then developed for solving the above mixed integer problem. Specifically, the optimal solution is first solved for an approximation problem which includes a subset of the original constraints. Stop if the current optimal solution satisfies the most restrict constraint defined in the paper or resolve the problem with additional constraints, otherwise. Two algorithms for computing the coefficients of the cutting plane are then provided, a forward algorithm based on a binary scenario tree starting from an empty scenario and a backward algorithm based on a binary scenario tree starting from a scenario with all demands realized. The authors suggested sorting the demands in the non-decreasing order of $d_i \tilde{y}_i$, where $\tilde{y}_i$ is the optimal selection solution for the approximation problem. The algorithms are then extended to solve the corresponding multi-period selective newsvendor problem and single-period problem with piecewise-linear costs. For practical reason, a heuristic algorithm is also developed in which demands with parameters satisfying $\frac{S_i}{p_i E[x_i]} + v \leq p_i$ are selected and the optimal total order quantity is solved as in the classic newsvendor problem. Numerical analysis shows that this heuristic algorithm performs pretty well for large problems.

Taaffe, Chahar, and Tirumalasetty [46] extended the selective newsvendor problem in Taaffe, Geunes and Romeijn [47] to incorporate the risk attitude of the newsvendor. Demands are again assumed to be normally distributed with mean $\mu_i$ and variance $\sigma^2_i$ but independent of marketing effort. The newsvendor selects demand sources to serve and determines the size of the total order. The expected profit function $G(Q, y)$ is the same as the one formulated in Taaffe, Geunes and Romeijn [47] with functions $\mu_i(a_i)$ and $\sigma_i(a_i)$ replaced by constants $\mu_i$ and $\sigma_i$. The authors considered two possible objectives of a risk-averse selective newsvendor. One is to minimize the total demand variance while achieving a pre-specified expected profit level (or achieving a pre-specified expected net revenue level, the first term in the expected profit function formulated in Taaffe, Geunes and Romeijn [47]). The other is to minimize the probability of achieving profits below a pre-defined profit level $T$, i.e., $\min F_G(T)$, where $F_G(\cdot)$ is the distribution of profit. Simulation is used in this paper to solve the problem with
the latter objective. As $F_G(T)$ is a uni-modal function of $Q$ for a given $y$ according to the test results, the authors suggested the use of line search to locate the optimal order quantity and then choose the $y$ that yields the best objective value. A heuristic algorithm is also introduced to improve time efficiency and is showed to be a good approximation.

6 Conclusions and Implications

In this paper, we have summarized the extant contributions on the newsvendor problem for managing inventories. Our approach has been to extend the current analysis by examining contributions and potential extensions related to customer demand, supplier prices, buyer risk profiles, and the selective newsvendor setting. In general, our review leads us to conclude that there still exist a set of future research opportunities within each area.

References


[19] A. Federgruen and A. Heching. Combined pricing and inventory control under uncer-


[21] Y. Gerchak and D. Mossman. On the effect of demand randomness on inventories and 


[23] C.A. Ingene and M.E. Parry. Coordination and manufacturer profit maximization: The 


271, 1983.

tainty and quantity discount: Behavioral implications and a new solution procedure. 


The University of Rhode Island started to offer undergraduate business administration courses in 1923. In 1962, the MBA program was introduced and the PhD program began in the mid 1980s. The College of Business Administration is accredited by The AACSB International - The Association to Advance Collegiate Schools of Business in 1969. The College of Business enrolls over 1400 undergraduate students and more than 300 graduate students.

Mission

Our responsibility is to provide strong academic programs that instill excellence, confidence and strong leadership skills in our graduates. Our aim is to (1) promote critical and independent thinking, (2) foster personal responsibility and (3) develop students whose performance and commitment mark them as leaders contributing to the business community and society. The College will serve as a center for business scholarship, creative research and outreach activities to the citizens and institutions of the State of Rhode Island as well as the regional, national and international communities.

The creation of this working paper series has been funded by an endowment established by William A. Orme, URI College of Business Administration, Class of 1949 and former head of the General Electric Foundation. This working paper series is intended to permit faculty members to obtain feedback on research activities before the research is submitted to academic and professional journals and professional associations for presentations. An award is presented annually for the most outstanding paper submitted.