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JEL Classification: G11; G23; G33

Keywords: Growth Option, Future Pension Benefit, Pension Funding, Pension Risk

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Abstract

In this study, I derive the value of stockholders’ claim on a pension surplus and stockholders’ liability for a pension deficit in the post-ERISA regulatory environment. Based on that valuation, I develop a model of corporate pension policies in which sponsoring firms weigh contributions to their pension plan against the exercise of growth options in the allocation of limited financial resources. The model shows how corporate pension funding and asset allocation policies are shaped by the sponsoring firms’ characteristics, such as growth options, the marginal corporate tax rate, and regulatory variables such as the deficit reduction contribution rate, the variable-rate insurance premium, and the maximum possible fraction of operating assets that can be seized by the Pension Benefit Guaranty Corporation. I discuss the policy implications of the theoretical results.

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I. Introduction

The recent financial market crisis has brought a new attention to the funding adequacy of corporation-sponsored defined benefit pension plans. According to Financier (2009), the aggregate deficit in pension plans sponsored by S&P 1500 reached $409 billion in the midst of the 2008 market crash, a shock reversal from the $60 billion surplus seen a year earlier. Mercer (2013) reports that the aggregate pension deficit of S&P 1500 companies increased by $73 billion to a record year-end high of $557 billion as of December 2012 with a funded ratio of 74%. Much of the pension deficit might be ultimately funded by tax payers' money via the Pension Benefit Guarantee Corporation (PBGC). The PBGC estimates that its financial risk for potential termination of underfunded pension plans sponsored by financially weak corporations has increased to $295 billion in 2012, an amount that has continued to worsen since the economic downturn in 2008. According to a recent report (2013) by the U.S. Government Accountability Office (2013), the single-employer pension program continues to be designated as high risk since July 2003. Policy makers and Congress, therefore, are responsible for developing public policies and legislating laws that would encourage sponsoring corporations to practice sound pension policies. To meet such responsibilities, it is important to understand how sponsoring corporations shape their pension funding and asset allocation policies.

In this research, I examine corporate defined benefit (DB) pension policies in a new perspective. In particular, I view DB pension plans as an alternative investment project competing with growth opportunities for funding. This new perspective provides insight into how sponsoring firms make pension funding and asset allocation decisions. As long as I understand, this is the first study to examine a relation among sponsoring firm's growth opportunities, stockholders' claim/liability on pension plans, and corporate pension
policies. In this research, I develop a model of pension funding and asset allocation policies in the post-ERISA regulatory environment. In this model, a representative sponsoring firm has two alternative investment projects competing for limited financial resources: pension plans and growth options. As a result, the sponsoring firm weighs the risk-adjusted marginal expected return on pension funding against the risk-adjusted after-tax marginal expected return on growth options.

The risk-adjusted marginal expected return on pension funding consists of the tax saving and the risk-adjusted marginal expected return on the pension asset return discounted by the marginal pension surplus or deficit factor. Discounting of the pension surplus comes from the fact that overfunded pension assets are not entirely considered the sponsoring firm’s assets due to excise taxes and other restrictions on the conversion of overfunded pension assets into operating assets. Burrow, Scholes, and Menell (1983), and Thomas() suggest that sponsoring firms can withdraw excess pension assets from their pension plans without paying excise and corporate taxes “by making small reductions each year in amounts contributed to the plan.” Firms sponsoring an overfunded plan are exempt from the minimum funding contributions (MFCs) that are required for those with an underfunded plan. As a result, firms sponsoring an overfunded plan can save the MFCs every year until the pension surplus is depleted. The annual MFCs in a given year are determined by the pension benefits accrued in the previous year. The value of stockholders’ claim on excess pension assets, therefore, equals the present value of a stream of future pension benefits that can be covered by excess pension assets.

Discounting of the pension deficit results from the fact that firms sponsoring an underfunded plan are only required to "slowly" remedy the pension deficit. Sponsoring firms are obligated to make deficit reduction contributions (DRCs) to their plan and pay an
insurance premium to the Pension Benefit Guarantee Corporation (PBGC) every year until their plan becomes fully funded. The value of stockholders' liability for underfunded pension plans, therefore, equals the present value of a sequence of DRCs and insurance premiums paid every year until the pension deficit is remedied.

This study shows that the pension asset allocation is determined by the marginal pension surplus discount factor (MPSDF) for overfunded plans, and the expected cost of severe underfunding associated with plan termination for modestly underfunded plans. The reason is that MPSDF constitutes the reward for stock market investment, and the expected cost of severe underfunding comprises the downside risk of stock market investment. It is shown that the optimal pension asset allocation is a mix of risky and riskless assets with a sufficiently small MPSDF for overfunded plans, and with a sufficiently large expected cost of severe underfunding modestly underfunded plans. For a severely underfunded plan, however, the optimal pension asset allocation turns out to be the entire pension fund invested in the risky asset. The reason is that the firm with a severely underfunded plan has been already incurring the cost of severe underfunding, which is a sunk cost. I suggest that an exponentially rising expected cost of severe underfunding, which reflects better the reality, would discourage the sponsoring firm of a severely underfunded plan to make such an extreme allocation.

I discuss policy implications of the theoretical results. In particular, I evaluate effectiveness of deficit reduction contributions and the variable-rate insurance premium which does not vary with pension asset risk. I find that even the deficit reduction contribution rule tightened by the Pension Protection Act (PPA) of 2006 would not induce sponsoring firms to make more voluntary contributions to their underfunded pension plan. However, the variable-rate insurance premium would encourage sponsoring firms of an underfunded plan to
make more voluntary contributions. I suggest a simple structural change in the variable-rate insurance premium which would discourage sponsoring firms to underfund their pension plan. The model developed in this study produce several testable empirical implications. They include a negative relation between sponsoring firms' growth options and pension funding, and a relation between the time distribution of future pension benefits and pension funding among others.

The remainder of the paper is organized as follows. Section II reviews the literature. Section III derives the value of stockholders’ claim/liability on pension plans and develops a model of corporate pension funding and asset allocation decisions. Section IV and Section V discusses policy implications and empirical implications, respectively. Section VI sets forth the conclusions.

II. Related Literature

Firms sponsoring DBPPs promise to provide a fixed amount of retirement benefit to their employees based primarily on employee tenure, age, and salary. Required by the ERISA of 1974, firms are obligated to earmark a certain amount of assets to meet their pension obligations. These pension plan funds are generally invested in equities and fixed income securities. Firms are also required to make financial contributions to their pension funds based on certain formula specified by the law and tax code. Despite a decline in their relative importance, due to a shift to defined contribution plans over the last 30 years, DBPPs still account for an important part of private sector retirement plans in the US. About 20% of all public firms in Compustat, mostly large firms, have DBPPs, covering about 44 million US
workers and employees, and aggregate DBPP assets in the US private sector amounted to $2.7 trillion as of the end of 2012.1

Both academic researchers and practitioners have devoted considerable attention to the issue of optimal pension funding and asset allocation strategies for DBPP-sponsoring firms. Subsequent to the passage of the ERISA in 1974, several studies made important contributions to the corporate pension plan management. However, most of the previous studies focus on one particular aspect of pension funding such as pension insurance and tax exemption of pension contributions. Sharpe (1976), and Treynor (1977) illustrate that in the absence of taxes and given the structure of a fixed insurance premium charged by the PBGC, firms can maximize the put value (and accordingly maximize shareholder wealth) by employing a “mini-max” strategy in pension plan management, achieving a minimum level of pension funding while investing in a maximum level of risky assets. In contrast, Black (1980) and Tepper (1981) suggest a “max-mini” strategy from a tax benefit perspective, achieving a maximum level of plan funding and a minimum level of investment in low-tax risky assets. By contrast, in this study, the entire benefits pension funding provides for stockholders are incorporated into the corporate pension funding decision. Those benefits include tax saving, pension put, and stockholders' claim on the pension asset return.

Rauh (2006) finds that pension sponsors decrease capital investment in response to a reduction in financial resources caused by required pension contributions. Rauh's finding indirectly supports the premise of this study that pension funding competes with funding of growth opportunities for sponsoring firms' scarce financial resources. Rahu (2006), however,

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1 The data are published by Investment Company Institute. Total assets held in DBPPs represented 13.1% of total employer-sponsored plan assets (or $19.9 trillion) in 2012. For comparison, defined contribution pension plans held $5.4 trillion in assets, among which the largest portion of assets was held in 401(k) accounts ($3.6 trillion in assets).
does not investigate how sponsoring firms allocate available financial resources between voluntary pension pending and funding of other investment projects.

More recently, Love, Smith, and Wilcox (2011), LSW hereafter, contributes to the pension literature by investigating how government regulations can affect corporate pension decisions in a model in which sponsoring firms trade off the need to compensate employees for the financial risk in their pension benefit against the cost advantage that may be gained by exploiting underpriced pension insurance. Both LSW and this study show that government pension policies are major determinants of corporate pension funding and asset allocation policies. However, there are several important differences between LSW and this study. In LSW, sponsoring firms make the pension funding decision to minimize compensation costs. In this study, sponsoring firms view pension funding as an alternative investment project. Therefore, sponsoring firms weigh the marginal expected return on pension funding against that on funding growth opportunities. In LSW, employees receive only a partial pension benefit guarantee from the PBGC and are thus actively involved in the design of the pension benefit structure. This study takes the structure of pension benefits as a given, and implicitly assumes that employees receive a full promised pension benefit guarantee from the PBGC. According to the PBGC, in fact, few DC plan participants are affected by the statutory maximum benefit coverage.

In LSW, the corporate pension policies are driven by policy variables such as the minimum funding requirement, the maximum benefit guarantee level, the structure of the insurance premium, and financial distress costs. In their model, therefore, sponsoring firms would make the same pension decisions and thus maintain the roughly same pension funding level and asset composition in a given regulatory environment. In this study, the corporate pension policies are determined by the marginal expected return on growth opportunities and
stockholders' claim/liability on pension plans. As a result, sponsoring firms would make
different pension decisions depending on their growth opportunities, even if they operate in
the same regulatory environment.

In regard to the corporate pension asset allocation policy, several authors attempt to
explain why firms invest pension funds so heavily in risky assets. Most notably, Sundaresan
and Zapatero (1997), and Lucas and Zaldes (2006) develop a model in which firms can invest
in equity to hedge against increases in future pension benefits that are positively correlated
with the stock market. Frank (2002) finds that firms’ tax benefits are positively associated
with the percentage of their pension assets invested in bonds. Bergstresser, Desai, and Rauh
(2006) demonstrate that firms may inflate short-term earnings by increasing the assumed rate
of return on plan assets, which would, in turn, result in a greater share of plan assets invested
in equities. Rauh (2009) finds that firms with poorly funded pension plans and weak credit
ratings allocate a greater share of pension fund assets to safer securities, whereas firms with
well-funded pension plans and strong credit ratings invest more heavily in equity. This study
departs from current literature in determinants of corporate pension asset allocation policies.
In particular, this study finds that corporate pension asset allocation policies are determined
by stockholders' claim on pension surplus and costs related to severe pension underfunding.

III. The Model

The model describes the behavior of a representative firm promising a pension benefit
in the post-ERISA regulatory environment. Therefore, I take the existence of the DBPP and
the PBGC, the government pension insurer, as a given. I use the model to investigate two
decisions the firm makes: how much free cash flow to use to fund the pension plan in a given
year and how to allocate the pension fund among various risk classes of assets.
A. Setting

In the model, the firm seeks to maximize the value of stockholder equity. The firm is expected to operate in multiple periods. At the end of each period, say, a time \( t \), the firm is bestowed with a free cash flow \( (X_t) \) generated from the previous period’s operation and other assets in place. The free cash flow and other assets in place constitute total assets in place \( (A_t) \). Each period, the firm receives short-lived growth options \( (G_{t+1}) \) that will become worthless if not exercised immediately.\(^2\) The firm has no outstanding debt. It follows that the value of stockholder equity \( (V_t) \) at the end of a representative time period \( t \) is equal to

\[
V_t = A_t + G_{t+1}
\]

where \( A_t = X_t + \text{other assets in place} \).

The firm has two choices for the disposal of free cash flow \( X_t \): (1) to exercise a subset of or all current growth options and/or temporarily park it in a risky financial asset that I use to represent the aggregate stock market or (2) to make voluntary contributions to the pension plan. I use the terms a risky financial asset and the stock market interchangeably throughout this paper. The exercise of current growth options turns growth opportunities into operating assets. Such growth options-turned operating assets are assumed to last one period and to generate a gross return \( \bar{R}_{G}(y) \) with a mean \( \bar{R}_{G}(y) \) and a range \([0, \infty)\) for an amount \( y \) of capital investment.\(^3\) The mean return \( \bar{R}_{G}(y) \) is assumed to decrease in the amount of investment \( y \) (i.e., a diminishing return on investment). The operating assets in place are assumed to produce a gross (one plus) return of \( \bar{R}_A \), with a mean \( \bar{R}_A \) and a range \([0, \infty)\). The operating assets in place, a risky financial asset, and a riskless asset are assumed to produce a gross (one

\(^2\) The firm may expect a stream of growth options that will arrive in the future. All future growth options arriving after period \( t \), however, are assumed away for the sake of simplicity.

\(^3\) This assumption is made for the sake of simplicity. The assumption of short-lived growth options turned operating assets can be relaxed so that they last more than one period without changing the main results qualitatively.
plus) return of $\bar{R}_A$ with a mean of $\bar{R}_A$ and a range of $[0, \infty)$, $\bar{R}_M$ with a mean of $\bar{R}_M$ and a range of $[0, \infty)$, and $R_f (> 1)$, respectively.\(^4\) The corporate capital gain tax rate is assumed to be the same as the corporate income tax rate.

The firm is assumed to sponsor a DBPP and to be enrolled in the PBGC’s pension benefit insurance program. As a result, the firm possesses plan assets ($P_t$) and pension liabilities ($L_t$). The pension plan has two possible funding statuses: overfunded and underfunded. If the market value of plan assets exceeds the present value of pension liabilities, the pension plan is considered “overfunded.” Otherwise, it is considered “underfunded.”

**B. Stockholders’ Claim/Liability on the Pension Plan**

Stockholders’ claim on overfunded pension assets and their share of unfunded pension liabilities are complex and differ with firm characteristics and regulatory variables. In this section, I derive the value of stockholders’ claim on pension surplus and their liability for pension deficit.

*B1. Stockholders’ Claim on Pension Surplus*

The stockholders of a firm sponsoring an overfunded plan may be entitled to only a fraction of the excess pension assets. Existing law requires that the firm must pay an excise tax of up to 50% in addition to the normal 34% corporate tax or pay a lower excise tax and share the excess pension assets with the plan participants if the firm converts the excess pension assets into operating assets through a reversion. As a result, the reversion would potentially leave stockholders with only 16 cents for each dollar of reversion.

\(^4\) Without loss of generality, $\bar{R}_{G(y)}$, $\bar{R}_A$, and $\bar{R}_M$ are assumed to be stationary with the same mean and $R_f$ is the same for all time periods for the sake of simplicity.
As suggested by Burrow, Scholes, and Menell (1983), and Thomas (1989), sponsoring firms have an indirect way of moving excess pension assets back into operation without paying excise and corporate taxes. One of them is to reduce the amount contributed to the pension plan each year. The ERISA of 1974 and several rounds of subsequent legislation (i.e., the PPA of 1987, the Retirement Protection Act of 1994, and the PPA of 2006) require firms sponsoring an underfunded plan to make mandatory contributions that amount to the greater of the MFCs and the DRCs. The MFCs consist of new pension benefits accrued in the current year (the normal cost) and a fraction of the funding shortfall (currently 10%). The DRCs comprise the first year’s contribution of 18–30% of any underfunding and installment payments of the rest of the underfunding over a period of three to five years.

A firm sponsoring an overfunded plan, however, is exempt from those mandatory contributions. As a result, the firm can gradually convert excess pension assets into operating assets by applying the pension surplus toward the required minimum level of funding contributions until the pension surplus is depleted. As a result, the value of stockholders’ net claim on a pension surplus (NCPS) is equal to the discounted value of a stream of new pension benefits to be accrued in the future:

\[
NC_{t}^{PS} = \sum_{i=t+1}^{T-1} \frac{PB_{i}^{accr}}{R_{C}^{t-i}} + \frac{qPB_{a}^{accr}}{R_{C}^{t-1}} \equiv \lambda_{over} (P_{t} - L_{t})
\]

where

\[PB_{i}^{accr} = \text{expected new pension benefits to be accrued in time } i,\]

\[R_{C} = \text{the corporate cost of capital;}\]

\[q = \text{fraction of pension benefits to be funded immediately.}\]

\[\lambda_{over} = \text{overfunding discount factor.}\]

5 Burrow, Scholes, and Menell (1983) discuss two other methods: increasing the fraction of total employee compensation in the form of promised pensions and increasing early retirement benefits. Unlike the method considered in the present study, their two methods may not be easily implemented since the compensation package must be negotiated with employees.
\( T \) = final period in which new pension benefits are covered by remaining excess pension assets,

\( \varphi (0 < \varphi \leq 1) \) = fraction of the final period’s pension benefits that can be covered by remaining excess pension assets,

\( \lambda_{\text{over}} \) = discount factor for a pension surplus.

Notice from expression (2) that future pension benefits are discounted back at the corporate cost of capital \( R_c \) since the pension plan itself is not allowed to raise the external fund against its pension assets. Only the sponsoring firm can raise the external fund to cover pension benefits in the capital market against the sponsoring firm’s entire assets (i.e., both operating and pension assets). The final period \( T \) is determined by the relation \( P_T - L_T \leq 0 \).

Let \( PB_t^{paid} \) denote the pension benefits paid out from pension assets to retired employees in time \( t \), and \( \bar{R}_{PA} \) denote the long-term average annual gross expected return on pension assets.\(^6\) The expected value of pension assets at time \( T, P_T \), can be expressed as:

\[
P_T = (\prod_{j=t+1}^T \bar{R}_{PA,j})P_t - \sum_{i=t+1}^T (\prod_{j=i+1}^{T-1} \bar{R}_{PA,j})PB_i^{paid}
= \bar{R}_{PA}^{T-t}P_t - \sum_{i=t+1}^T \bar{R}_{PA}^{T-i} PB_i^{paid}
\]

The value of pension liabilities at time \( T \) consists of the future value of a stream of new pension benefits to be accrued from time \( t \) to \( T \) minus the future value of a stream of pension benefits paid out from time \( t \) to \( T \) plus the future value of pension liabilities outstanding as of time \( t \). Reflecting the actual practice, pension liabilities are assumed to grow at the gross riskless rate \( R_f \) so that The value of pension liabilities at time \( T \) can be expressed as:

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\(^6\) As the pension surplus becomes smaller in size over time, the sponsoring firm may rebalance pension assets and change the asset composition which would, in turn, change the return on pension assets.
\[ L_T = R_f^{T-t}L_t + \sum_{i=t+1}^{T} R_f^{T-i}PB_i^{accr} - \sum_{i=t+1}^{T} R_f^{T-i}PB_i^{paid} \]  

The relation between the pension surplus \((P_t - L_t)\) at time \(t\), a stream of new pension benefits to be accrued, and a stream of pension benefits paid out from time \(t\) to \(T\) can be expressed as:

\[ P_t - L_t = \sum_{i=t+1}^{T} \frac{R_f^{T-i}}{\hat{R}_{PA}^{T-t}} PB_i^{accr} + \frac{\phi}{\hat{R}_{PA}^{T-t}} PB_i^{accr} + \sum_{i=t+1}^{T} \frac{R_f^{T-i}}{\hat{R}_{PA}^{T-t}} PB_i^{paid} + \left( \frac{R_f^{T-t}}{\hat{R}_{PA}^{T-t}} - 1 \right) L_t \]  

Notice from Expressions (2) and (5) that \( \lambda_{over} = 1 \) if \( R_C = \hat{R}_{PA} = R_f \); and \( \lambda_{over} < 1 \) if \( R_C > \hat{R}_{PA} = R_f \). In a more plausible case where \( R_C > \hat{R}_{PA} > R_f \), \( \lambda_{over} \) can be greater or less than one, depending on \( R_C, \hat{R}_{PA}, \) and \( R_f \). The reason that \( \lambda_{over} \) can be greater than one is that pension assets are assumed to grow in value faster than pension liabilities so that the expected pension surplus increases over time.

**B2. Stockholder Liability for Pension Deficit**

As discussed before, firms of an underfunded pension plan are obligated to make the greater of MFCs and DRCs to the pension plan every year until it becomes fully funded. To reflect the structure of tax-deductible DRCs imposed on a firm sponsoring an underfunded plan, I assume that the firm is required to contribute a constant fraction, \( k, 0 < k < 1 \), of the pension deficit to the pension fund every year until the pension deficit is remedied. In exchange for providing insurance coverage, the PBGC requires firms sponsoring an underfunded plan to pay a variable-rate premium (currently $9 per $1,000 of underfunding) in addition to a fixed premium of $35 per participant. Let \( m \) denote the variable-rate premium as the percentage of unfunded pension liabilities. Notice that similar to pension benefits, the pension deficit can be financed only against the sponsoring firm's entire assets. The value of stockholders’ net liability for a pension deficit \((NL^{PD})\) can then be expressed as
$$NLP_t^{PD} = (1 - \tau)(k + m) \sum_{i=1}^{\infty} (1 - k)^{-i} \left( \frac{\hat{R}_{t+i}^t P_t - R_{t+i}^t L_t}{R_C} \right)$$

$$\simeq (1 - \tau)(k + m) R_C \left[ \frac{P_t}{R_C - (1-k)\hat{R}_{PA}} - \frac{L_t}{R_C - (1-k)R_f} \right]$$

$$\equiv \lambda_{under} (P_t - L_t) \quad (6)$$

where $\lambda_{under}$ is the discount factor for pension deficit. Notice from expression (6) that if $R_C = \hat{R}_{PA} = R_f$ and $m = 0$, $\lambda_{under} = 1 - \tau$, which means that sponsoring firms are liable only for a fraction of the pension deficit.

The ERISA of 1974 and several rounds of later legislation also state that if firms sponsoring an underfunded plan fail to make the minimum required contributions and the unpaid amounts total more than $1$ million, the PBGC can perfect and enforce a statutory lien on the firm’s property, up to 30% of its net worth. The net worth is widely interpreted as the fair market value of equity of the sponsoring firm, excluding pension assets and liabilities. As a result, the “augmented” pension assets consist of the pension plan assets plus a fraction of the firm’s stockholder equity without pension assets and liabilities. It follows that the value of stockholders’ net claim on the augmented pension assets ($NC_{t}^{APA}$) is:

$$NC_{t}^{APA} = \max\{ \phi V^{NP}_{t} + \lambda_{under} (P_t - L_t), 0 \} \quad (7)$$

where

$\phi =$ fraction of the intrinsic value of the firm’s equity that can be seized by the PBGC in the event of the firm’s failure to make required contributions,

$$V^{NP}_{t} = A_t + G_{t+1}.$$ 

Taken together, the value of stockholders’ equity of the firm at the end of time $t$ can be expressed as
\[ V_t = (1 - \phi)(A_t + G_{t-1}) + \max\{\phi V_t^{NP} + \lambda_{PS}(P_t - L_t), 0\} \]  

(8)

where

\[ \lambda_{PS} = \begin{cases} 
\lambda_{over} & \text{if } P_t \geq L_t, \\
\lambda_{under} & \text{if } P_t < L_t.
\end{cases} \]

C. Pension Funding Decision

At the beginning of every period, the sponsoring firm repeats the same pension funding and asset allocation decisions. The pension funding decision involves allocation of the free cash flow \((X_t)\) between the exercise of current growth options and contribution to the pension fund. The pension asset allocation decision entails allotment of the pension fund between risky and riskless financial assets. In this section, I analyze the firm’s pension funding decision, given pension asset allocation. I consider two alternative circumstances: with internal financing and with external financing.

C1. With Internal Financing

This subsection analyzes the firm’s pension funding decision with internal financing. Let \(\alpha, 0 \leq \alpha \leq 1\), denote the fraction of the free cash flow \(X_t\) that the firm allocates to the exercise of current growth options and/or investment in the stock market. The remainder \(1 - \alpha\) is allocated to the pension fund. This study focuses on the pension funding decision in a given period and not a dynamic cash flow allocation decision between current and future growth options. The sponsoring firm’s operating assets in place are assumed to generate a steady stream of cash flow sufficient to fund growth options and the pension plan every period. If the firm chooses to contribute part or all of the free cash flow to the pension fund,
the firm receives a one-time tax credit on the amount contributed. At the beginning of every period, the firm repeats the same pension funding decision to maximize the market value of stockholder equity:

\[ \text{Max}_{0 \leq \alpha \leq 1} V_t = R_C^{-1} E_t[V_{t+1}] = R_{OA}^{-1}(1 - \phi) E_t[\tilde{R}_i(A_i - X_t)] + \int_0^{\alpha X_t} \tilde{R}_G(y) dy + R_f^{-1}(1 - \phi)(1 - \alpha)X_t \]

(operating assets in place) (growth options turned operating assets) (corporate tax paid) (tax saving from pension contributions)

\[ + E_t[R_N^{-1}\phi\tilde{R}^{NP}_{t+1} + R_P^{-1}\lambda \tilde{R}_{PA}(P_t + (1 - \alpha)X_t) - L_{t+1})\{\phi\tilde{R}^{NP}_{t+1} + \{\tilde{R}_{PA}(P_t + (1 - \alpha)X_t) - L_{t+1}\} \geq 0] \]

(stockholders' claim on augmented pension assets)

(9)

where

- \( R_{OA}^{-1} \) = the discount rate for the sponsoring firm's operating assets,

- \( R_{PP}^{-1} \) = the discount rate for the pension plan,

- \( R_{NP}^{-1} \) = the discount rate for the sponsoring firm's non-pension assets (that consists of operating assets and tax benefit from pension contributions),

\[ \tilde{R}_{PA} = \text{gloss (one plus) return on pension assets, i.e., } \tilde{R}_{PA}(\alpha \tilde{R}_M) \equiv R_f + \alpha(\tilde{R}_M - R_f) \]

- \( \omega (0 \leq \omega \leq 1) \) = fraction of plan assets invested in the risky financial asset.

The sponsoring firm weighs the discounted (risk-adjusted) marginal expected after-tax return on current growth options against the discounted (risk-adjusted) marginal expected return on a pension contribution. The marginal expected after-tax return on current growth options is a decreasing function of \( \alpha \) due to the diminishing return on investment. The marginal expected return on a pension contribution consists of the discounted marginal
corporate tax rate and the discounted expected tax-free return on plan assets. Notice from expression (9) that the return made by plan assets is not subject to the corporate tax, while the return earned by the exercise of current growth options is taxable. The exemption of the return on pension assets from the corporate tax constitutes another benefit of a pension contribution in addition to its tax deductibility.

Let \( \alpha, 0 \leq \alpha \leq 1 \), denote the allocation ratio at which \((1 - \alpha)X_t\) is equal to the maximum possible tax-deductible contribution a firm can make to an overfunded pension plan.\(^7\) With no tax benefit available, the firm would rather use the free cash flow to exercise current growth options and/or invest in the stock market. Therefore, a firm sponsoring an overfunded plan would allocate at least \( \alpha X_t \) to exercise current growth options and/or invest in the stock market.

**Proposition 1**

Suppose that the objective function (9) is concave in \( \alpha \). Then the optimal \( \alpha^* \), where \( \alpha^* \in (\alpha, 1) \) for an overfunded plan and \( \alpha^* \in (0, 1) \) for an underfunded plan, is determined at which

\[
R_{OA}^{-1} \left[ \bar{R}_G(\alpha^*) - \pi(\bar{R}_G(\alpha^*) - 1) - \text{Prob}(a) \phi(E_a(R_G(\alpha^*)) - \pi(E_a(R_G(\alpha^*) - 1)) \right] = R_f^{-1} \tau + R_{PP}^{-1} [\text{Prob}(b) MPSDF* E_b(\bar{R}_{PA}) + \text{Prob}(c) MPDDF* E_c(\bar{R}_{PA})]
\]

where \( MPSDF \) (Marginal Pension Surplus Discount Factor) \( \equiv (R_C^{T(\alpha^*)-(t+1)})^{-1} \),

\( MPDDF \) (Marginal Pension Deficit Discount Factor) \( \equiv (1 - \tau)(k + m)(\frac{R_C}{R_C - (1-k)R_{PA}}) \),

\( a \equiv \bar{P}_{t+1} - L_{t+1} < -\phi\bar{N}_{t+1} \),

\(^7\) The DBPP-related laws set the maximum tax-deductible contributions beyond which the firm of an overfunded plan is not allowed to receive a tax credit. Furthermore, the firm may be subject to an excise tax on contributions above the maximum. The Retirement Protection Act of 1994 and the PPA of 2006 state that the maximum tax-deductible contributions may not exceed the greatest of (1) the MFCs, (2) the amount necessary to fully fund the plan’s current liability, and (3) the normal cost, plus the plan’s past service cost with amortization periods reduced to 10 years.
\[ b \equiv \bar{P}_{t+1} - L_{t+1} \geq 0, \]
\[ c \equiv -\phi V^{NP}_{t+1} \leq \bar{P}_{t+1} - L_{t+1} < 0, \]

or \( \alpha^* = \alpha \) or 1 for an overfunded plan and \( \alpha^* = 0 \) or 1 for an underfunded plan.

**Proof:** See Appendix.

For ease of understanding the results of Proposition 1, I take a simpler case, where stockholders’ claim on the augmented pension assets is always positive. In such a case, the optimal \( \alpha^* \) is determined at which

\[
R^{-1}_{OA} \left[ \bar{R}_G (\alpha^*) - \tau (\bar{R}_G (\alpha^*) - 1) \right]
= R^{-1}_f + R^{-1}_{PA} \left[ \text{Prob}(b)MPSDF*E_b(\bar{R}_{PA}) + \text{Prob}(c) MPDDF* E_c(\bar{R}_{PA}) \right] \tag{10}
\]

Note from expression (10) that the optimal \( \alpha^* \) is determined at which the risk-adjusted marginal expected after-tax return on growth options equals the discounted marginal corporate tax rate plus a weighted average of the risk-adjusted expected tax-free returns on pension assets discounted by the marginal discount factors for overfunded and underfunded plans, with the weights being the probabilities of the pension plan becoming overfunded and underfunded due to pension contributions and pension asset return. Expression (10) indicates that an additional dollar of voluntary contribution to the pension plan would enable the sponsoring firm to save the marginal corporate tax, and increase stockholders' claim on excess pension assets or reduce the burden of making mandatory pension contributions by the

---

8 Since the firm can temporarily park the free cash flow in the stock market, \( \bar{R}_G (\alpha^*) \geq \bar{R}_M \). Otherwise, \( \alpha^* \) is determined at which \( R^{-1}_{OA} \left[ \bar{R}_M - \tau (\bar{R}_M - 1) - \text{Prob}(a) \phi [E_a(R_G(\alpha^*)) - \tau (E_a(R_G(\alpha^*)) - 1)] =
R^{-1}_f \tau + R^{-1}_{PA} \left[ \text{Prob}(b)MPSDF*E_b(\bar{R}_{PA}) + \text{Prob}(c) MPDDF* E_c(\bar{R}_{PA}) \right] \) and \( \alpha^* \) is divided between current growth options and the risky financial asset. The division of \( \alpha^* \) is determined at which \( \bar{R}_G (\alpha) - \tau (\bar{R}_G (\alpha) - 1) = \bar{R}_M - \tau (\bar{R}_M - 1) \). Since this study focuses on the pension funding decision, I assume that \( \bar{R}_G (\alpha^*) \geq \bar{R}_M \) for simplicity.
expected tax-free return on plan assets discounted by the marginal discount factors. The marginal pension surplus discount factor (\textit{MPSDF}) measures the present value of one dollar of the pension benefit to be accrued in the farthest future which is covered by an additional dollar of voluntary contribution. The marginal pension deficit discount factor (\textit{MPDDF}) measures the after-tax fractional DRC and variable-rate insurance premium to be reduced by an addition dollar of voluntary contribution.

In the case where stockholders’ claim on the augmented pension assets is negative, the sponsoring firm would surrender up to a fraction (currently 30%) of stockholder equity. In such an event, stockholders of the sponsoring firm and the PBGC would share the marginal expected after-tax return on current growth options. If the pension deficit is so large that even the augmented pension assets fails to cover it, it would be in the best interest of stockholders for the sponsoring firm to walk away from the pension plan (i.e., file a distress termination), laying all pension obligations on the PBGC.\footnote{If a sponsoring firm issues a notice of intent to terminate its DBPP, the PBGC examines the firm’s financial condition to determine whether the sponsoring firm satisfies one of several financial distress tests. The financial distress tests include whether it has been demonstrated that the sponsor or affiliate cannot continue in business unless the plan is terminated. If the PBGC approves of the application for distress termination, the sponsoring firm and each controlled group member are jointly and severally liable to the PBGC under ERISA Section 4062(b) for the total amount of unfunded benefit liabilities determined in accordance with 29 CFR 4022 Subpart D. If the PBGC determines that a plan does not qualify for distress termination, the plan will remain ongoing under close monitoring by the PBGC. Bereskin (2009) notes that sponsoring firms may be permitted to terminate their plans only when filing for bankruptcy protection since the passage of the Single-Employer Pension Plan Amendment Act of 1986.}

For both overfunded and underfunded pension plans, several common factors determine voluntary pension contributions that, in turn, determine pension funding. Those factors include the firm-specific characteristics such as the expected return on current growth options, the marginal corporate tax rate, and the time distribution of future pension benefits. For both overfunded and underfunded plans, a greater marginal expected return on current growth options leads to lower pension funding. The effect of the marginal corporate tax rate,
however, is not the same for overfunded and underfunded plans. In the case of an underfunded plan, unlike for an overfunded pension plan, an increase in the marginal corporate tax rate would not necessarily result in more voluntary pension contributions. While the sponsoring firm receives an immediate tax credit from a pension contribution, an additional dollar contribution to an underfunded plan would reduce stockholders’ net pension liability only by one minus the marginal corporate tax rate. The reason is that the tax deductibility of pension contributions has been already factored into the stockholders’ net pension liability.

**Corollary 1**

$$\alpha^* \in (\alpha, 1) \text{ decreases in } MPSDF, \text{ i.e., } (R_{C}^{(t)} - (t+1))^{-1}.$$  

Proof: See Appendix.

Pension funding is also partially determined by the time distribution of future pension benefits through $MPSDF$. It should be noticed that $MPSDF$ depends on how far in the future the last one dollar of excess pension assets is used to cover the accrued pension benefits. As a result, if the pension surplus is too large in size or few pension benefits are expected to be accrued in the near future, the marginal expected return on pension contribution would be small relative to that on growth options. In such a case, the firm would rather invest the entire free cash flow in current growth options and/or the stock market. In the other extreme case, if a large amount of pension benefits is expected to be accrued in the imminent future, the marginal expected return on pension contribution would surpass that on growth options. In such an event, the sponsoring firm would contribute the entire free cash flow to the pension fund.

**Corollary 2**

$$\alpha^* \text{ decreases in } k \text{ or } m.$$  

Proof: See Appendix.
For an underfunded pension plan, the pension funding is determined by $MPDDF$ which is, in turn, determined by the fraction of the funding shortfall, $k$, required to be made up every year, and the variable-rate premium, $m$. A higher $k$ or $m$ would increase $MPDDF$, which, in turn, raises the marginal expected return on pension contributions, and, thus, would lead to a greater allocation of the free cash flow to the pension fund.

**Corollary 3**

$\alpha^*$ decreases in $\phi$.

Proof: See Appendix.

An increase in $\phi$, the maximum possible fraction of stockholder equity that can be seized by the PBGC, would raise the upper limit of the pension deficit for which stockholders of the sponsoring firm are liable. Making stockholders responsible for a greater share of unfunded pension liabilities would induce the sponsoring firm to make more voluntary contributions to the pension plan.

**C2. With External Financing**

This subsection extends the analysis of the corporate pension funding decision to the case where the firm relies on external financing. For simplicity, the firm is assumed to retain no free cash flow at the end of time $t$. The firm can borrow an external fund $F$ at a gross interest rate $R_D(F)$, with $R_D = R_f$ in the case of no default risk. The interest rate $R_D(F)$ increases in $F$ due to a positive relation between $F$ and default risk. Since the PBGC’s claim receives a lower priority in the event of default, debt claims are assumed to be senior to pension liabilities.\(^\text{10}\) With external financing, objective function (9) is modified as follows:

\(^\text{10}\)Keating (1993) argues that the PBGC’s arguments in favor of priority status in the event of a sponsoring firm's bankruptcy are weak. Citing several references, Bereskin (2009) notes that "the PBGC’s claims are typically relegated to that of other unsecured claims [in the event of sponsoring firms' bankruptcy]." Schroeder (2005) reports that the PBGC recovers only 7% of its claim of unfunded pension liabilities in stockholder equity of the reorganized firm.
\[ \text{Max}_{F, \theta \geq 0} V_i = R_C^{-1} E^t_\lambda [\tilde{V}_{t+1}] \]

\[ = (1 - \phi) E_t [R_{\theta A}^{-1} \{ \tilde{G}, A_t \} + \int_0^{af} \tilde{R}_G(y)dy - R_D(F)F - \tau_i(\tilde{G}_i - 1)A_i, \]

\[ + \int_0^{af} (\tilde{R}_G(y) - 1)dy - (R_D(F) - 1)F \} + R_f^{-1} \tau(1 - \alpha)F] \tilde{G}_i A_t + \int_0^{af} \tilde{R}_G(y)dy - RD(F)F \geq 0] \]

\[ + E_t [R_{NP}^{-1} \phi \tilde{V}_{t+1}^{NP} + R_{PP}^{-1} \lambda_{PS} \{ \tilde{R}_{PA}(P_i + (1 - \alpha)F) - L_{t+1} \} | \phi \tilde{V}_{t+1}^{NP} + \{ \tilde{R}_{PA}(P_i + (1 - \alpha)F) - L_{t+1} \} \geq 0] \]

\[(11)\]

where

\[ R_D(F) = \text{gross external financing cost as a percentage of the amount of external} \]

\[ \text{financing, with } R_D'(F) \geq 0, \]

\[ F = \text{amount of external financing}. \]

Figure 1 demonstrates the sponsoring firm’s pension funding decision. The optimal amount of external financing is determined for the amount at which the greater of the discounted marginal expected after-tax return on current growth options and the discounted marginal expected return on a pension contribution is equal to the discounted marginal after-tax cost of external financing. Once the optimal amount of external financing is determined, the optimal allocation of the external fund between current growth options and the pension plan is determined in the same manner as with internal financing. The only difference is that with external financing, the marginal expected returns on current growth options and pension contributions are conditional on the sponsoring firm remaining solvent.
D. Pension Asset Allocation Decision

This section analyzes the pension asset allocation decision. For a given pension funding decision, the sponsoring firm’s objective function for pension asset allocation is reduced to:

\[
\text{Max}_{0 \leq \omega \leq 1} \, \mathbb{E}[R^{-1}_{NP} \phi \tilde{N}_{t+1}^P + R^{-1}_{PP} \lambda PS \{ \tilde{R}_{PA} (P_t + (1 - \alpha)X) - L_{t+1} \}] \geq 0
\]  

(12)

As discussed in Section III C, the objective function (12) indicates that the sponsoring firm is responsible for a pension deficit up to a fraction \( \phi \) of the sponsoring firm's stockholder equity, excluding pension assets and liabilities.

It has been so far assumed that there is no additional cost associated with a severe pension deficit on the part of the sponsoring firm. With a significant proportion of plan assets invested in the stock market, a market correction could cause plan assets to fall significantly below pension liabilities in value. The PPA of 2006 requires employers with at least one plan that is less than 80% funded to report annually additional information so that the PBGC can closely monitor the situation. Therefore, the sponsoring firm would bear an additional cost of meeting reporting requirements and being monitored by the PBGC. Furthermore, an increase in pension deficit would raise the probability of pension plan termination initiated by the PBGC, which involves a plan termination-related cost on the part of the sponsoring firm. Upon plan termination, sponsoring firms must pay a termination premium of $1,250 per participant in the year of termination and in each of following two years (established by the Deficit Reduction Act of 2005). In addition, sponsoring firms have to bear various types of direct and indirect costs associated with
plan termination, including the transaction costs of disposing of operating assets up to 30% of the equity value (if the unpaid amount of MFCs exceeds $1 million). Bicksler and Chen (1985) also discuss the legal expenses arising from lawsuits associated with plan termination and the costs resulting from poor labor relations.

In light of these observations, I assume that the sponsoring firm with a severe pension deficit incurs a percentage cost \( c \) proportional to the pension deficit exceeding a certain threshold. I then modify stockholders’ net liability for a pension deficit as follows:

\[
NL^P_t \equiv \lambda_{\text{under}}(P_t - L_t) = (1 - \tau)(k + m)R_t\frac{P_t}{R_{C(1-k)\bar{R}_P}} - \frac{L_t}{R_{C(1-k)R_f}} \text{ if } \delta L_t \leq P_t \leq L_t;
\]

\[
(1 - \tau)(k + m + c)R_t\frac{P_t}{R_{C(1-k)\bar{R}_P}} - \frac{L_t}{R_{C(1-k)R_f}} \text{ if } P_t < \delta L_t
\]

where \( c \) is a constant with \( c > 0 \) and \( \delta \) is a constant with \( 0 < \delta < 1 \).

In the following analysis, the objective function (12) is assumed to be concave in \( \omega \).

**Proposition 2**

1. For an overfunded pension plan (i.e., \( R_t(P_t + (1 - \alpha)X_t) \geq L_{t+1} \)), there exists a critical marginal pension discount factor, \( MPSDF^* \), such that for \( MPSDF < MPSDF^* \), the optimal pension asset allocation is a mix of risky and riskless financial assets (i.e., \( 0 < \omega^* < 1 \)) and, for \( MPSDF \geq MPSDF^* \), \( \omega^* = 1 \).

2. For a modestly underfunded pension plan (i.e., \( \delta L_{t+1} \leq R_t(P_t + (1 - \alpha)X_t) < L_{t+1} \)), there exists a critical \( c^* \) such that, for \( c > c^* \), the optimal pension asset allocation is a mix of risky and riskless financial assets (i.e., \( 0 < \omega^* < 1 \)) and, for \( c \leq c^* \), \( \omega^* = 1 \).

Proof: See Appendix.
The intuition behind this result is following. The sponsoring firm weighs benefits of allocating more pension assets toward the risky financial asset against costs. For an overfunded pension plan, a high $MPSDF$ means a relatively large amount of pension benefits to be accrued in the near future, toward which the sponsoring firm can apply the pension surplus. With a sufficiently high $MPSDF$, the firm with an overfunded pension plan would invest the entire pension fund in the risky financial asset since stockholders can claim most of the incremental excess pension assets resulting from a stock market appreciation. Therefore, a sufficient condition for a mix of risky and riskless assets is a sufficiently low $MPSDF$. It is a remote possibility, however, that a market correction causes an overfunded pension plan to turn into severely underfunded. Therefore, the cost of severe underfunding is not relevant to the asset allocation decision for an overfunded plan.

For a modestly underfunded plan, a greater allocation of pension assets toward the risky financial asset would provide the sponsoring firm with the opportunity for curtailing DRCs and saving the insurance premium by reducing the pension deficit through a capital gain. But it could also cause the firm to incur the expected cost of severe underfunding. As the firm raises the percentage of stock market investment, the marginal expected gain (i.e., $MPDDF$) remains constant, but the marginal expected cost of severe underfunding increases since the probability of falling into a severely underfunded status rises with the percentage of stock market investment. With a sufficiently large cost of severe underfunding, therefore, the firm would find a mix of risky and riskless assets optimal. It is unlikely, however, that the underfunded plan turns into overfunded as a result of more stock market investment. As a result, for an underfunded plan, $MPSDF$ is not a factor that determines the asset allocation decision.
In contrast, the firm with a severely underfunded plan, has been already incurring the cost of severe underfunding, which is a sunk cost. As a result, benefits of stock market investment would outweigh its costs. In such a circumstance, the firm with a severely underfunded plan may find it optimal to invest the entire pension assets in the risky financial asset. We have assumed a linear relationship between the pension deficit and the cost of severe underfunding. It is more plausible, however, that the cost of severe underfunding increases at an increasing rate with the size of pension deficit. With such an exponential relationship, even the firm with a severely underfunded plan would prefer to take a more conservative approach in asset allocation.

The following comparative statics result is obtained.

**Corollary 1**

\[ \omega^* \text{ increases in } MPSDF_{t+1}, \text{ i.e., } (R_{C_t}^{T(P_t-L_{t+1})+(r+1)})^{-1}. \]

Proof: See Appendix.

The intuition behind this result is the same as discussed above for Proposition 2.

The effect of \( k \) and \( m \) on the optimal pension asset allocation, \( \omega^* \), is vague for modestly and severely underfunded pension plans. It can be shown, however, that \( \omega^*, 0 < \omega^* < 1 \), decreases in \( m \) under mild conditions. The reason is that a higher \( m \) would increase stockholders' liability for pension deficit. As a result, the firm would reduce allocation of pension assets to the risky financial assets in response to a higher insurance premium.

**Corollary 2**

\( \omega^*, 0 < \omega^* < 1, \text{ decreases in } \phi. \)

Proof: See Appendix.
An increase in \( \phi \) would cause the sponsoring firm to lose more of its operating assets to the PBGC if a stock market correction makes the pension plan severely underfunded. It follows that a higher \( \phi \) would induce the sponsoring firm to invest less in the risky financial asset.

**IV. Policy Implications**

Previous sections show that for overfunded and modestly underfunded plans, pension funding and asset allocation decisions are influenced by \( MPSDF \), \( MPDDF \), which is, in turn, determined by policy variables such as the DRC rate \( (k) \), the variable-rate insurance premium \( (m) \), and the fraction of the firm’s operating assets that can be seized by the PBGC \( (\phi) \). This section evaluates the effectiveness of these policy variables in influencing corporate pension funding and asset allocation policies.

The model implicitly assumes that sponsoring firms are required to fully fund their underfunded pension plans within a certain period. Prior to the PPA of 2006, however, sponsoring firms were generally required to fund only 90% of pension liabilities and could delay funding unfunded pension liabilities through accounting maneuvering for as many as 30 years (Brown (2008)). Such a partial funding requirement can be easily incorporated into expression (6), the valuation of stockholders’ share of unfunded pension liabilities. It can be shown that the partial funding requirement has the same effect on pension funding and asset allocation decisions as does the DRC rule.

The PPA of 2006 gradually phased in a 100% funding target and tightened the DRC rule such that sponsoring firms are required to remedy their pension deficits within four to seven years. I examine whether the tightened DRC rule can induce sponsoring firms to alter the funding and asset allocation strategies with the following plausible numerical example. For the numerical example, I set \( \text{Prob}(c) \) (the probability of the plan being modestly
underfunded) = 1. \, k = 0.50, \, m = 0.009, \, R_j = 1.05, \, \hat{R}_{PA} = 1.12, \, R_C = 1.20, \, \tau = 0.35,
reflecting evidence that sponsoring firms made an average annual return of 12% on their pension assets in the period of 1989-2007. For those parameter values, $MPDDF$ is estimated to be 0.62. If we apply the estimate of 0.620 for $MPDDF$ to expression (10), the right-hand side of expression (10) equals 1.0448. This means that sponsoring firms of a modestly underfunded plan make a meager 4.48% of return on each one dollar contributed to the pension plan. Such a relatively low return may explain why an overwhelming majority of sponsoring firms made no voluntary contributions to their pension fund at all in the period of 1989-2007. In light of the estimate above, we expect a majority of sponsoring firms to continue to make no voluntary contributions with the current DRC rule. The new DRC rule mandated in the PPA of 2006 would certainly shorten the period in which underfunded pension plans should be fully funded. However, the numerical example above suggests that the new DRC rule would not prevent sponsoring firms from underfunding their pension plan in the first place. The reason is that the reward for voluntary contributions to the pension plan are relatively low.

The PBGC charges sponsoring firms a fixed premium of $35 per participant (as of 2011), plus 0.9% of any unfunded vested benefits. The variable-rate premium, which is set by Congress, does not vary with pension asset risk, and, as a result, is widely believed to be below the economically fair level. However, it would be practically impossible for the insurance premium to be set commensurate with pension asset risk due to a high measurement and monitoring cost. This study shows that a higher variable-rate premium ($m$) would induce sponsoring firms of an underfunded plan to fund their plan more. The current structure of the insurance premium, therefore, is effective in inducing sponsoring firms to improve their pension status, even if the premium is not designed to reflect pension asset risk.
Furthermore, this study shows that $k$ and $m$ are interchangeable and have the same effect on the corporate pension decisions. The DRC rule has been already tightened and it may not be practically possible to tighten it further. This study suggests that policy makers can adjust the variable-rate premium to obtain the same effect on the corporate pension funding policy as altering the DRC rule.

This study also suggests that sponsoring firms of a severely underfunded plan would choose to invest the entire pension fund in the stock market regardless of $k$ or $m$. The pension put (i.e., the ability of sponsoring firms to walk away from their pension plan by surrendering up to 30% of the equity value) makes such an incentive stronger. As discussed in the previous section, an exponentially increasing cost of severe underfunding may dampen such an incentive. However, such a cost structure alone may not be sufficient enough to prevent sponsoring firms from taking an excessive pension asset risk. When sponsoring firms are bullish on the stock market, they may find the expected return on the stock market outweigh the expected cost of plan termination when making a pension asset allocation decision. A flat variable-rate premium rate is currently imposed on unfunded pension liabilities regardless of the size of the pension deficit. This study suggests that if the variable-rate premium is structured such that sponsoring firms of larger unfunded pension liabilities are more heavily penalized, they would have less incentive to underfund the pension plan. For instance, the variable-rate pension premium rate can be designed to be progressive so that a higher premium rate is charged to the sponsoring firms with a larger pension deficit.

The results discussed in Section III suggest that the maximum possible fraction of sponsoring firms’ operating assets that can be seized by the PBGC, $\phi$, is an effective policy tool that influences sponsoring firms’ pension funding and asset allocation policies. A higher $\phi$ would encourage sponsoring firms to make more voluntary contributions to their pension
plan and allocate more of pension funds toward safe assets. However, a change of \( \phi \) requires approval from Congress, which is a long process. A possible remedy for such a protracted process is that Congress provides the PBGC with the authority to change \( \phi \) within a certain range without Congressional approval.

V. Empirical Implications

In this section, I discuss empirical implications of the theoretical results discussed in section III.

Implication 1: Young firms with ample growth options have lower pension funding ratio than mature firms with limited growth options.

Young firms may have more growth opportunities than mature firms since mature firms may have already exhausted most of their growth options. As a result, young firms with ample growth opportunities may prefer to use financial resources for exercise of growth options rather than pension funding.

Implication 2: Sponsoring firms with a higher MPSDF maintain a relatively higher pension funding ratio.

Proposition 1 shows that MPSDF is a determinant of pension funding and a higher MPSDF would lead to more pension funding. MPSDF is determined by the size of pension surplus and the time distribution of pension benefits to be accrued in the future. Specifically, MPSDF is negatively related to the size of pension surplus and positively related to the amount of pension benefit to be accrued in the imminent future. As a result, MPSDF may be estimated as follows:

\[
MPSDF_{i,t} = 1 \text{ if } PS_{i,t-1} \leq PB_{i,t};
\]
\[ \text{MPSDF}_{i,t} = R_C^{-1} \text{ if } PB_{i,t} < PS_{i,t-1} \leq PB_{i,t} + R_C^{-1}PB_{i,t+1}; \]

\[ \text{MPSDF}_{i,t} = R_C^{-2} \text{ if } PB_{i,t} + R_C^{-1}PB_{i,t+1} < PS_{i,t-1} \leq PB_{i,t} + R_C^{-1}PB_{i,t+1} + R_C^{-2}PB_{i,t+2}; \]

\[ \text{MPSDF}_{i,t} = \min(R_C^{-3}, \frac{PB_{i,t} + R_C^{-1}PB_{i,t+1} + R_C^{-2}PB_{i,t+2}}{PS_{i,t-1}}) \quad \text{if } PB_{i,t} + R_C^{-1}PB_{i,t+1} + R_C^{-2}PB_{i,t+2} < PS_{i,t-1} \]

**Implication 3:** For overfunded pension plans, pension funding is positively related to the marginal corporate tax rate. For underfunded pension plans, the tax effect is weak.

For an overfunded plan, a higher marginal corporate tax rate would enable the sponsoring firm to save more tax from pension contributions. For an underfunded plan, however, an increase in the marginal corporate tax rate has two opposite effects. It would allow the sponsoring firm not only to save more tax but also to reduce stockholders' claim on the pension asset return by the marginal corporate tax rate. The corporate tax effect, therefore, is weak for underfunded plans.

**Implication 4:** The firm with a low credit rating or a high debt ratio maintains lower pension funding relative to pension liabilities.

With the pension plan financed by an external fund, the cost of external financing is another factor for the pension funding decision. Other things being equal, the firm with a lower credit rating or a larger debt outstanding would incur a higher cost of external financing. With a high cost of external financing, pension funding is more likely to produce a negative return. Carroll and Niehaus (1998) report evidence on the impact of unfunded pension liabilities and excess pension assets on corporate debt ratings. This study suggests that such causality may run the other way around. That is, the pension plan is underfunded simply because of a high
borrowing cost which results from a low credit rating. Such an external financing constraint would eventually lead to pension plan underfunding.  

**Implication 5:** *The firm with a higher MPSDF takes more pension asset risk.*

Proposition 2 and Corollary 1 show that the firm with a higher MPSDF has an incentive to take more pension asset risk since the firm can apply the excess pension assets increased by a capital gain toward the required pension contributions.

**VI. Summary and Conclusion**

In this study, pension funding is viewed as an alternative investment project and the sponsoring firm weighs pension funding against funding of growth opportunities. I derive the value of stockholders’ claim on overfunded pension assets and liability for unfunded pension liabilities. Based on that valuation, I develop a model of corporate pension funding and asset allocation policies in the post-ERISA regulatory environment. This model shows how corporate pension policies are shaped by the sponsoring firm’s characteristics, such as growth opportunities and marginal corporate tax rate, and policy variables such as minimum funding requirements, the variable-rate insurance premium, and the maximum possible fraction of a firm’s operating assets that can be seized by the PBGC. I also discuss the policy and empirical implications of the theoretical results. As an extension of this study, I am currently testing the empirical implications and find that the preliminary test results strongly support the empirical predictions.

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11 Similar theoretical results are also provided by Cooper and Ross (2002).
Appendix

Proof of Proposition 1: Objective function (9) can be simplified as:

\[ V_t = R^{-1}_{OA} \left[ \bar{G}(A_t - X_t) + \int_0^{\alpha_t} \bar{G}_t(y) \, dy - \tau \{ \int_0^{\alpha_t} (\bar{G}_t(y) - 1) \, dy \} \right] + R^{-1}_T \pi(1 - \alpha)X_t \]

\[ - R^{-1}_{OA} \text{Prob}(\alpha) \phi_E_c \left( \bar{G}_{t+1}^{NP} \right) + R^{-1}_{PP} \text{Prob}(b) \lambda_{\text{over}} \left[ E_b(\bar{R}_{PA}) (P_t + (1 - \alpha)X_t) - L_{t+1} \right] \]

\[ + R^{-1}_{PP} \text{Prob}(c) \lambda_{\text{under}} \left[ E_c(\bar{R}_{PA}) (P_t + (1 - \alpha)X_t) - L_{t+1} \right] \]  

(A1)

where \( \lambda_{\text{over}}[E_b(\bar{R}_{PA}) (P_t + (1 - \alpha)X_t) - L_{t+1}] = E_b(\bar{R}_{PA}) \left( \sum_{i=t+1}^{T} \frac{P_{B_i}}{R_{C_i}^{(t+1)}} + \frac{\phi_P B_T}{R_{C_i}^{(t+1)-\alpha(t+1)}} \right) \).

For \( \phi < 1 \), differentiating (A1) with respect to \( \alpha \) and dividing by \( X_t \) yields

\[ \frac{dV_t}{d\alpha} (1/X_t) = (1 - \text{Prob}(a)\phi) \left[ R^{-1}_{OA}(\bar{G}_t(\alpha) - \tau(\bar{G}_t(\alpha) - 1)) \right] - R^{-1}_T \tau \]

\[ - R^{-1}_{PA} \left[ \text{Prob}(b) \cdot MPSDF \ast E_b(\bar{R}_{PA}) + \text{Prob}(c) \cdot MPDDF \ast E_c(\bar{R}_{PA}) \right] \]

(A2)

where \( MPSDF \equiv \left( R_{C}^{-\alpha(t+1)} \right)^{-1} \); \( MPDDF \equiv (1 - \tau)(k + m)(\frac{R_{C}}{R_{C} - (1-k)R_{PA}}) \).

since \( \frac{d\lambda_{\text{over}}(P_{t+1} - L_{t+1})}{d\alpha} = \frac{d(\sum_{i=t+1}^{T} \frac{P_{B_i}}{R_{C_i}^{(t+1)}})}{d(P_{t+1} - L_{t+1})} \cdot \frac{d(P_{t+1} - L_{t+1})}{d\alpha} \), and \( \frac{d(P_{t+1} - L_{t+1})}{d\alpha} = d(PB_{t+1} + \ldots) \)

For \( \phi = 1 \), expression A(1) is not differentiable with respect to \( \alpha \). In that case, objective function (9) reaches a local maximum at a discontinuity point where \( (1 - \text{Prob}(a)\phi) \left[ R^{-1}_{OA}(\bar{G}_t(\alpha) - \tau(\bar{G}_t(\alpha) - 1)) \right] \)

intersects with

\( (1 - \text{Prob}(a)\phi) R^{-1}_T \tau + R^{-1}_{PP} \left[ \text{Prob}(b) \cdot MPSDF \ast E_b(\bar{R}_{PA}) + \text{Prob}(c) \cdot MPDDF \ast E_c(\bar{R}_{PA}) \right] \).
If expression \((A2) > 0\) for \(\alpha = \alpha^*\) for an overfunded plan and for \(\alpha = 0\) an underfunded plan, and expression \((A2) < 0\) for \(\alpha = 1\), objective function (9) reaches a local maximum at \(\alpha^* \in (\alpha, 1)\) for an overfunded plan and at \(\alpha^* \in (0, 1)\) for an underfunded plan where expression \((A2) = 0\) since objective function (9) is concave in \(\alpha\) by assumption. If expression \((A2) > 0 \ \forall \ \alpha, 0 < \alpha < 1\), \(\alpha^* = 1\). If expression \((A2) < 0 \ \forall \ \alpha, 0 < \alpha < 1\), \(\alpha^* = 0\) for an underfunded plan. Q.E.D.

**Proof of Corollary 1 of Proposition 1:** Notice from (A2) that for \(\alpha = \alpha^*\), where \(0 < \alpha^* < 1\),

\[
(1 - \text{Prob}(a)\phi)[R_G^{-1}(\tilde{R}_G(\alpha) - \pi(\tilde{R}_G(\alpha) - 1)) - R_f^{-1}\tau] \\
- R_{PP}^{-1}[\text{Prob}(b)*MPSDF*E_b(\tilde{R}_{PA}) - \text{Prob}(c)*MPDDF*E_c(\tilde{R}_{PA})] = 0
\]

(A3)

Differentiating (A3) with respect to \(MPSDF \equiv (R_C^{T(\alpha)} - (t+1))^{-1}\) yields:

\[
\frac{\partial (A3)}{\partial (R_C^{T(\alpha)} - (t+1))^{-1}} = - R_{PP}^{-1}\text{Prob}(b)*E_b(\tilde{R}_{PA}) < 0.
\]

It follows that

\[
\frac{d\alpha^*}{d(R_C^{T(\alpha)} - (t+1))^{-1}} = - \frac{\partial (A3)}{\partial \alpha^*} < 0 \text{ since } \frac{\partial (A3)}{\partial \alpha^*} < 0 \text{ by assumption. Q.E.D.}
\]

**Proof of Corollary 2 of Proposition 1:** Differentiating (A3) with respect to \(k\) produces:

\[
\frac{\partial (A3)}{\partial k} = - R_{PP}^{-1}\text{Prob}(c)[(\frac{(1 - \tau)R_C}{R_C - (1-k)\tilde{R}_{PA}} - (1 - \tau)(k+m)R_C\tilde{R}_{PA})]E_c(\tilde{R}_{PA}) < 0 \text{ since}
\]

\[
R_C(R_C - (1-k)\tilde{R}_{PA}) - (k+m)R_C\tilde{R}_{PA} = R_C(R_C - (1-m)\tilde{R}_{PA}) > \min(mR_C^2, mR_C\tilde{R}_{PA}) > 0.
\]

It follows that
\[
\frac{d\alpha^*}{dk} = - \frac{\partial (A3)}{\partial k} < 0 \quad \text{since} \quad \frac{\partial (A3)}{\partial \alpha^*} < 0 \quad \text{by assumption.}
\]

Differentiating (A3) with respect to \(m\) produces:

\[
\frac{\partial (A3)}{\partial m} = - R_{PP}^{-1} \text{Prob}(c) \left( \frac{(1-\tau)R_C}{R_C - (1-k)R_{PA}} \right) E_c(\bar{R}_{PA}) < 0 \quad \text{since} \quad R_C \geq \bar{R}_{PA} \quad \text{by assumption.}
\]

It follows that

\[
\frac{d\alpha^*}{dm} = - \frac{\partial (A3)}{\partial m} < 0 \quad \text{since} \quad \frac{\partial (A3)}{\partial \alpha^*} < 0 \quad \text{by assumption.} \quad \text{Q.E.D.}
\]

**Proof of Corollary 3 of Proposition 1:** Differentiating (A3) with respect to \(\phi\) produces:

\[
\frac{\partial (A3)}{\partial \phi} = - \text{Prob}(a)\left[ R_{OA}^{-1}(\bar{R}_G(\alpha) - \pi(\bar{R}_G(\alpha) - 1)) - R_f^{-1}\tau \right] < 0 \quad \text{since} \quad \alpha = \alpha^*, \ (1 - \text{Prob}(a)\phi) [R_{OA}^{-1}(\bar{R}_G(\alpha) - \pi(\bar{R}_G(\alpha) - 1)) - R_f^{-1}\tau] - R_{PP}^{-1}[\text{Prob}(b)*MPSDF*E_b(\bar{R}_{PA}) - \text{Prob}(c)*MPDDF*E_c(\bar{R}_{PA})] \]

= 0 and thus it must be true that \(R_{OA}^{-1}(\bar{R}_G(\alpha) - \pi(\bar{R}_G(\alpha) - 1)) > R_f^{-1}\tau\). It follows that

\[
\frac{d\alpha^*}{d\phi} = - \frac{\partial (A3)}{\partial \phi} < 0 \quad \text{since} \quad \frac{\partial (A3)}{\partial \alpha^*} < 0 \quad \text{by assumption.} \quad \text{Q.E.D.}
\]

**Proof of Proposition 2:** Define \(M_{t+1} \equiv P_t + (1 - \alpha)X_t\). Objective function (13) can be rewritten as:

Max\(\omega \ E_t\left[ R_{PP}^{-1}\phi\tilde{V}_{t+1}^{NP} + R_{PP}^{-1}\lambda_{PS}\{ \tilde{R}_{PA}(P_t + (1 - \alpha)X_t) - L_{t+1}\}|\phi\tilde{V}_{t+1}^{NP} + \{ \tilde{R}_{PA}(P_t + (1 - \alpha)X_t) - L_{t+1}\} \geq 0 \right]

= E_t\left[ R_{PP}^{-1}\phi\tilde{V}_{t+1}^{NP} + R_{PP}^{-1}\lambda_{over}\{(R_f + \alpha(\bar{R}_M - R_f))M_{t+1} - L_{t+1}\}|\bar{R}_M \geq x \right]

+ E_t\left[ R_{PP}^{-1}\phi\tilde{V}_{t+1}^{NP} + R_{PP}^{-1}\lambda_{over}\{(R_f + \alpha(\bar{R}_M - R_f))M_{t+1} - L_{t+1}\}|y \leq \bar{R}_M < x \right]

+ E_t\left[ R_{PP}^{-1}\phi\tilde{V}_{t+1}^{NP} + R_{PP}^{-1}\lambda_{under}\{(R_f + \alpha(\bar{R}_M - R_f))M_{t+1} - L_{t+1}\}|z \leq \bar{R}_M < y \right]

+ E_t\left[ R_{PP}^{-1}\phi\tilde{V}_{t+1}^{NP} + R_{PP}^{-1}\lambda_{under}\{(R_f + \alpha(\bar{R}_M - R_f))M_{t+1} - L_{t+1}\}|z \leq \bar{R}_M < y \right]
\[
\begin{align*}
&= \int_0^\infty \int_0^\infty \left[ R_{pp}^{-1} \phi V_{t+1}^{NP} + R_{pp}^{-1} \lambda_{\text{over}} \{ (R_f + \omega(R_M - R_f))(M_{t+1} - L_{t+1}) \} dF(R_M, V_{t+1}^{NP}) \right] \\
&\quad + \int_0^\infty \int_0^\infty \left[ R_{pp}^{-1} \phi V_{t+1}^{NP} + R_{pp}^{-1} \lambda_{\text{under}} \{ (R_f + \omega(R_M - R_f))(M_{t+1} - L_{t+1}) \} dF(R_M, V_{t+1}^{NP}) \right] \\
&\quad + \int_0^\infty \int_0^\infty \left[ R_{pp}^{-1} \phi V_{t+1}^{NP} + R_{pp}^{-1} \lambda_{\text{under}} \{ (R_f + \omega(R_M - R_f))(M_{t+1} - L_{t+1}) \} dF(R_M, V_{t+1}^{NP}) \right]
\end{align*}
\]

(A4)

where

\[
x = R_f + \omega^{-1}\left( \frac{L_{t+1}}{M_{t+1}} - R_f \right), \quad y = R_f + \omega^{-1}\left( \frac{\partial L_{t+1}}{M_{t+1}} - R_f \right), \text{ and } z = \max[0, R_f + \omega^{-1}\left( \frac{L_{t+1} - \Phi V_{t+1}^{NP}}{M_{t+1}} - R_f \right)].
\]

Differentiating expression (A4) with respect to \( \omega \) by applying the Leibniz integral rule yields:

\[
\frac{\partial (A4)}{\partial \omega} = M_{t+1}R_{pp}^{-1}\left( (R_{C}^{-1})_{t+1} \right) \int_0^\infty \left[ (R_M - R_f) dG(R_M) \right] \\
\quad + \frac{(1-\tau)(k + m)R_C}{R_{C} - (1-k)\hat{R}_{PA}} \int_0^\infty \left[ (R_M - R_f) dF(R_M, V_{t+1}^{NP}) \right] \\
\quad + \frac{(1-\tau)(k + m + c)R_C}{R_{C} - (1-k)\hat{R}_{PA}} \int_0^\infty \left[ (R_M - R_f) dF(R_M, V_{t+1}^{NP}) \right]
\]

(A5)

since \( \lambda_{\text{over}}(P_{t+1} - L_{t+1}) = \sum_{i=t+1}^{t+T} \frac{PB_i}{R_{C}^{-1}(t+1)} \) and \( \frac{d\lambda_{\text{over}}(P_{t+1} - L_{t+1})}{d\omega} = \frac{d(\sum_{t+1}^{t+T} \frac{PB_i}{R_{C}^{-1}(t+1)})}{d(P_{t+1} - L_{t+1})} = \frac{1}{R_{C}^{-1}(t+1)} \) and

\[
\frac{d\left( P_{t+1} - L_{t+1} \right)}{d\omega} = \frac{d(\sum_{t+1}^{t+T} \frac{PB_i}{R_{C}^{-1}(t+1)})}{d(P_{t+1} - L_{t+1})} = (P_{B_{t+1}} + \ldots + \frac{d\phi_{B_{t+1}}}{R_{C}^{-1}(t+1)}) / d(P_{t+1} - L_{t+1}) = \frac{1}{R_{C}^{-1}(t+1)} \text{ and}
\]

\[
\frac{d\left( P_{t+1} - L_{t+1} \right)}{d\omega} = M_{t+1} (R_M - R_f), \text{ and } \lambda_{\text{under}}(P_{t+1} - L_{t+1}) = (1-\tau)(k + m) \frac{R_{C}^{-1} P_{t+1}}{R_{C}^{-1}(1-k)\hat{R}_{PA}}.
\]
\[
\frac{R_{C_t+1}}{R_C - (1-k)R_f} \text{ and } \frac{d \lambda_{\text{under}}(P_{t+1} - L_{t+1})}{d \omega} = \frac{d((1-\tau)(k+m)R_{C_{t+1}})}{R_C - (1-k)R_{PA}} \times \frac{dP_{t+1}}{d\omega} \quad \text{and} \quad \frac{dP_{t+1}}{d\omega} = M_{t+1}(R_M - R_f).
\]

1. For an overfunded plan (i.e., \(R_fM_t \equiv R_f(P_t + (1 - \alpha)X_t) > L_{t+1}\)),

\[\text{(A5)}_{\omega < 0(x=0)} = M_{t+1}R_{PP}^{-1} \left( R_C^{T-(i+1)} \right)^{-1} \int_0^\infty (R_M - R_f) dG(R_M) > 0 \quad \text{for} \quad \omega \leq \omega(x = 0) \quad \text{since} \quad \tilde{R}_M > R_f.\]

For \(\omega = 1\),

\[
\frac{\partial (A5)}{\partial \omega} = M_{t+1}R_{PP}^{-1} \left[ \left( R_C^{T-(i+1)} \right)^{-1} \int_{R_f}^{\infty} (R_M - R_f) dG(R_M) + (R_C^{T-(i+1)})^{-1} \int_0^{R_f} (R_M - R_f) dG(R_M) \right]
\]

\[
+ \frac{(1-\tau)(k+m)R_C}{R_C - (1-k)\tilde{R}_{PA}} \int_0^x (R_M - R_f) dF(R_M, V_{t+1}^{NP})
\]

\[
+ \frac{(1-\tau)(k+m+c)R_C}{R_C - (1-k)\tilde{R}_{PA}} \int_0^y (R_M - R_f) dF(R_M, V_{t+1}^{NP})
\]

\[
(\text{A6})
\]

where \(x = \frac{L_{t+1}}{M_{t+1}}, y = \frac{\delta L_{t+1}}{M_{t+1}}, \) and \(z = \max[0, \frac{L_{t+1} - \phi V_{t+1}^{NP}}{M_{t+1}}].\)

Notice from (A6) that (A6) < 0 for \((R_C^{T-(i+1)})^{-1} = 0\) and (A6) decreases in \((R_C^{T-(i+1)})^{-1}\).

Therefore, there exists a critical MPSDF* such that for MPSDF < MPSDF*, (A6) < 0. Since (A5) > 0 for \(\omega\) close to zero and (A6) < 0 for \(\omega = \text{MPSDF} < \text{MPSDF*}\), and objective function (13) is concave in \(\omega\) by assumption, \(0 < \omega* < 1\) for MPSDF < MPSDF*, and \(\omega* = 1\) for MPSDF \(\geq\) MPSDF*.

2. For an underfunded plan (i.e., \(R_fM_t \equiv R_f(P_t + (1 - \alpha)X_t) < L_{t+1}\), as \(\omega\) approaches zero,
\[
\lim_{\omega \to 0} (A5) = \int_{0}^{\infty} \left( R_{M} - R_{f} \right) dF(R_{M}, V_{i+1}^{NP}) > 0
\]

since \( \lim_{\omega \to 0} x = \infty, \lim_{\omega \to 0} y = -\infty \) and \( \bar{R}_{M} > R_{f} \).

For \( \omega = 1 \),

\[
\frac{\partial (A5)}{\partial \omega} = M_{i+1} R_{PP}^{-1} \sum_{x}^{\infty} \int_{0}^{y} (R_{M} - R_{f}) dF(R_{M}, V_{i+1}^{NP}) + (1 - \tau)(k + m)R_{C} \sum_{x}^{\infty} \int_{0}^{y} (R_{M} - R_{f}) dF(R_{M}, V_{i+1}^{NP})
\]

\[
+ \frac{(1 - \tau)(k + m + c)R_{C}}{R_{C} - (1 - k)\bar{R}_{PA}} \sum_{x}^{\infty} \int_{0}^{z} (R_{M} - R_{f}) dF(R_{M}, V_{i+1}^{NP}) \]

(A7)

where \( x = \frac{L_{i+1}}{M_{i+1}}, y = \frac{\phi L_{i+1}}{M_{i+1}}, \) and \( z = \max[0, \frac{L_{i+1} - \phi V_{i+1}^{NP}}{M_{i+1}}] \).

Notice that \( \frac{(1 - \tau)(k + m + c)R_{C}}{R_{C} - (1 - k)\bar{R}_{PA}} \sum_{x}^{\infty} \int_{0}^{y} (R_{M} - R_{f}) dF(R_{M}, V_{i+1}^{NP}) < 0 \) since \( y < 1 < R_{f} \) and, thus, (A7) decreases in \( c \). Therefore, there exists a critical \( c^{*} \) such that for \( c > c^{*} \), (A7) < 0. Since (A5) > 0 for \( \omega \) close to zero and (A5) < 0 for \( \omega = 1 \) and \( c > c^{*} \), and objective function (13) is concave in \( \omega \) by assumption, \( 0 < \omega^{*} < 1 \) for \( c > c^{*} \), and \( \omega^{*} = 1 \) for \( c \leq c^{*} \). Q.E.D.

**Proof of Corollary 1 of Proposition 2:** Notice from (A5) that for \( \omega = \omega^{*}, 0 < \omega^{*} < 1 \),

\[
M_{i+1} R_{PP}^{-1} \sum_{x}^{\infty} \int_{0}^{y} (R_{M} - R_{f}) dF(R_{M}, V_{i+1}^{NP}) + \frac{(1 - \tau)(k + m)R_{C}}{R_{C} - (1 - k)\bar{R}_{PA}} \sum_{x}^{\infty} \int_{0}^{y} (R_{M} - R_{f}) dF(R_{M}, V_{i+1}^{NP})
\]

\[
+ \frac{(1 - \tau)(k + m + c)R_{C}}{R_{C} - (1 - k)\bar{R}_{PA}} \sum_{x}^{\infty} \int_{0}^{z} (R_{M} - R_{f}) dF(R_{M}, V_{i+1}^{NP}) \] = 0

(A8)
where

\[ x = R_f + \omega^{-1} \left( \frac{L_{t+1}}{M_{t+1}} - R_f \right), y = R_f + \omega^{-1} \left( \frac{\delta L_{t+1}}{M_{t+1}} - R_f \right), \text{ and } z = \max[0, R_f + \omega^{-1} \left( \frac{L_{t+1} - \theta V_{t+1}}{M_{t+1}} \right) - R_f] . \]

Differentiating expression (A8) with respect to \((R_C - (t+1))^{-1}\) yields

\[ \frac{\partial(A7)}{\partial(R_C - (t+1))^{-1}} = M_{t+1} R_{PP}^{-1} \int_x^\infty (R_M - R_f) dG(R_M) > 0 \text{ for all } x \text{ since } R_M > R_f. \]

It follows that

\[ \frac{d\omega^*}{d(R_C - (t+1))^{-1}} = - \frac{\partial(A7)}{\partial(R_C - (t+1))^{-1}} > 0 \text{ since } \frac{\partial(A8)}{\partial\omega} < 0 \text{ by assumption. Q.E.D.} \]

**Proof of Corollary 2 of Proposition 2:** If \( \phi \tilde{V}_{t+1} + \{ \tilde{R}_{P,j} (P_i + (1 - \alpha)Y_{i,t}) - L_{t+1} \} \geq 0 \) for all \( \tilde{V}_{t+1} \), then (A5) at \( \omega = \omega^* \) is equal to:

\[
(R_C - (t+1))^{-1} \int_x^\infty (R_M - R_f) dG(R_M) + \frac{(1 - \tau)(k + m)R_C}{R_C - (1 - k)\tilde{R}_{PA}} \int_0^\infty (R_M - R_f) dF(R_M, V_{t+1}^{NP}) \\
+ \frac{(1 - \tau)(k + m + c)R_C}{R_C - (1 - k)\tilde{R}_{PA}} \int_0^\infty (R_M - R_f) dF(R_M, V_{t+1}^{NP}) = 0
\]

(A9)

Differentiating (A9) with respect to \( k \) yields:

\[ \frac{\partial(A9)}{\partial k} = - \frac{(1 - \tau)m}{k^2} \int_0^\infty (R_M - R_f) dF(R_M, V_{t+1}^{NP}) - \frac{(1 - \tau)(m + c)}{k^2} \int_0^\infty (R_M - R_f) dF(R_M, V_{t+1}^{NP}) \]

\( > 0 \text{ since } \tilde{R}_M = R_f \text{ and } \frac{(1 - \tau)m}{k^2} < \frac{(1 - \tau)(m + c)}{k^2} \text{ and } y < R_M. \) It follows that

\[ \frac{d\omega^*}{dk} = - \frac{\partial(A9)}{\partial\omega^*} > 0 \text{ since } \frac{\partial(A8)}{\partial\omega^*} < 0 \text{ by assumption.} \]
Differentiating (A8) with respect to $m$ generates:

$$
\frac{\partial (A8)}{\partial k} = \frac{1-\tau}{k} \int_{0}^{\infty} (R_{M} - R_{f}) dF(R_{M}, V_{t+1}^{NP}) + \frac{1-\tau}{k} \int_{0}^{\infty} (R_{M} - R_{f}) dF(R_{M}, V_{t+1}^{NP}) < 0
$$

since $\bar{R}_{M} = R_{f}$ and $x < \infty$. It follows that

$$
\frac{d\omega^{*}}{dm} = -\frac{\partial (A8)}{\partial m} < 0 \text{ since } \frac{\partial (A8)}{\partial \omega^{*}} < 0 \text{ by assumption. Q.E.D.}
$$

**Proof of Corollary 2**: Differentiating (A5) with respect to $\phi$ produces:

$$
\frac{\partial (A5)}{\partial \phi} = \frac{(1-\tau)(k+m+c)R_{C}}{R_{C} - (1-k)\hat{R}_{P4}} \int_{0}^{\infty} v_{t+1}^{NP} (z - R_{f}) dF(R_{M} = z, V_{t+1}^{NP}) < 0 \text{ since } z < R_{f}. \text{ It}
$$

follows that

$$
\frac{d\omega^{*}}{d\phi} = -\frac{\partial (A5)}{\partial \phi} < 0 \text{ since } \frac{\partial (A5)}{\partial \omega^{*}} < 0 \text{ by assumption. Q.E.D.}
$$
References

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Figure 1: Pension Funding Decision with External Financing

For both overfunded and underfunded pension plans, the optimal amount of external financing is determined at the amount where the greater of the marginal expected after-tax return on current growth options and the marginal expected return on a pension contribution is equal to the marginal after-tax cost of external financing. Once the amount of external financing is determined, the optimal allocation of the external fund between current growth options and pension contributions is determined at the ratio where the marginal expected after-tax return on current growth options is equal to the marginal return on pension contributions. For simplicity, the following graphs illustrate the optimal external financing and pension funding decisions with no possibility of bankruptcy or augmented pension assets falling below pension liabilities in value. I define \( b \equiv \hat{p}_{t+1} - L_{t+1} \geq 0 \) and \( c \equiv \hat{p}_{t+1} - L_{t+1} < 0 \).

Case 1: An external fund is raised.

\[
\begin{align*}
\bar{R}_G, \ R_D \\
R_G^{-1}(\bar{R}_G(\alpha) - \tau(\bar{R}_G(\alpha) - 1)) \\
R_f^{-1} \tau + R_P \hat{b}(\text{Prob}(b))\text{MPSDF}_b(\bar{R}_PA) + \text{Prob}(c) \text{MPDDF}_c(\bar{R}_PA) \\
R_d(F) - \tau(R_d(F) - 1) + (1-\tau)R'_d(F)F
\end{align*}
\]
Case 2: No external fund is raised.

\[ R_D(F) - \tau R_D(F) + (1-\tau)R_D'(F)F = R_D^{-1} \left( \frac{\bar{R}_G(\alpha) - \tau(\bar{R}_G(\alpha) - 1)}{\bar{R}_D(t)} \right) \]

\[ R_D^{-1} \left( \frac{\bar{R}_G(\alpha) - \tau(\bar{R}_G(\alpha) - 1)}{\bar{R}_D(t)} \right) = \frac{R_D^{-1} \left( \frac{\bar{R}_G(\alpha) - \tau(\bar{R}_G(\alpha) - 1)}{\bar{R}_D(t)} \right)}{R_D^{-1}} \]

\[ R_D^{-1} \left( \frac{\bar{R}_G(\alpha) - \tau(\bar{R}_G(\alpha) - 1)}{\bar{R}_D(t)} \right) = \frac{R_D^{-1} \left( \frac{\bar{R}_G(\alpha) - \tau(\bar{R}_G(\alpha) - 1)}{\bar{R}_D(t)} \right)}{R_D^{-1}} \]

\[ R_D^{-1} \left( \frac{\bar{R}_G(\alpha) - \tau(\bar{R}_G(\alpha) - 1)}{\bar{R}_D(t)} \right) = \frac{R_D^{-1} \left( \frac{\bar{R}_G(\alpha) - \tau(\bar{R}_G(\alpha) - 1)}{\bar{R}_D(t)} \right)}{R_D^{-1}} \]

\[ F^* = 0 \]
Founded in 1892, the University of Rhode Island is one of eight land, urban, and sea grant universities in the United States. The 1,200-acre rural campus is less than ten miles from Narragansett Bay and highlights its traditions of natural resource, marine and urban related research. There are over 14,000 undergraduate and graduate students enrolled in seven degree-granting colleges representing 48 states and the District of Columbia. More than 500 international students represent 59 different countries. Eighteen percent of the freshman class graduated in the top ten percent of their high school classes. The teaching and research faculty numbers over 600 and the University offers 101 undergraduate programs and 86 advanced degree programs. URI students have received Rhodes, Fulbright, Truman, Goldwater, and Udall scholarships. There are over 80,000 active alumnae.

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