Design and Pricing of a Multi-function Product

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Abstract

In the electronics product market, multifunction products (MFPs, or fusion products) are often offered after some single-function products (SFPs) have gained popularity and been accepted by consumers. Some consumers who own one or several distinct SFPs will decide to upgrade to purchase the MFP. Some consumers, who own none of these SFPs, will decide to purchase the MFP instead of the SFP. This paper investigates the optimal MFP quality design and pricing decisions based on individual consumer’s utility from the perspective of the manufacturer. We assume two distinct SFPs have been offered to the market first. The consumers will purchase each SFP when its utility from purchasing is positive. Given that some consumers have owned one or two SFPs, the firm then decides the quality design and the price of the MFP to maximize its profit from offering the MFP.

Our results show that the MFP should be priced with some discount to entice those consumers who own one or two SFPs to ‘upgrade’ to the MFP. We also investigate the impact of several parameters on the optimal decisions on four different segments of consumers. We identify three fusion strategies: diminishing, additive and synergistic, which are moderated by the MFP’s optimal compound quality level and the quality levels of two SFPs. The numerical examples provide the preferred conditions of three fusion strategies and sensitivity of parameter changes on key variables (pricing, quality level, and the derived demand from four segments).

1 Introduction

In recent years, multi-function products (MFPs) proliferate in the electronics market place since they offer multiple functionalities typically not offered by single-function products (SFPs). Other advantages of MFPs include space saving, data synchronization across various functions, lower total cost, and a “fashion” premium (Avery, 2004; Magid, 1998; Schonfeld, 2004). As MFPs are often launched after SFPs are in the market, we observe that many buyers purchase an MFP after owning one or more SFPs. The manufacturers of electronic products view this switching or upgrade as a great potential for a continuous revenue stream (Gilroy, 2007) and invest a lot of capital in MFP’s design in order to maintain market leadership.
From a technology perspective, continuous developments in storage capacity, processors, and wireless transmission speed, lead to an increased rate of introduction of upgraded models. An MFP is often a faster, relatively cheaper (in terms of the unit price of a quality attribute), and a well integrated product that provides high levels of functionality and thus, attracts many buyers. According to a report released by Rubicon Consulting, Inc. in April 2009, more than half of iPhone users are iPod owners and about 50% of iPhone users purchase iPhone to replace their conventional mobile phones\(^1\).

From a product innovation/invention perspective, the availability of a new functionality is typically introduced through an SFP. As the market matures, the SFP becomes more of a “commodity,” and hence, firms start to offer more integrated devices with multiple functionalities. For example, through the use of digital technologies, music quickly became available in various audio formats. As a result, the Apple iPod which provides access to digital music quickly gained popularity and can be regarded as a “commodity” product in this setting. In an attempt to capitalize on customer preferences for multiple functionalities, manufacturers today offer MFP’s which integrate digital music playback capabilities (e.g., cell phones with digital music playback function such as Apple’s iPhone and Sony Ericsson’s W800 Walkman cellular phone).

Several factors influence the product offering decision of the MFP manufacturer. First is the *fusion technology* that integrates several functions together. Current technologies in microprocessor and circuit board design makes fusing several functions into one device an easy task. However, the intrinsic synergy among these functions and the degree of seamless integration primarily decide how well the market will accept the MFPs. Second, the “quality” levels, and the price of current SFP should be also considered. Not all MFPs become successful. While integrating a TV and DVD player may be logical, the TV-DVD player combo is still a niche product (Chen & Carrillo, 2010). One possible reason is that the price of this combination product is not necessarily lower than the sum of the prices of a stand-alone TV and a stand-alone DVD player. Further, the combination product only provides a minimal quality improvement in wire connectivity. Third, the quality levels and the prices of the MFPs determine product demand and hence, firm profitability. The fusion technology takes time to mature and its improvement comes from the earlier generations in the market. For example, after several previous generations, Palm’s Treo 650 still has some design flaws

\(^1\)http://rubiconconsulting.com/downloads/whitepapers/Rubicon-iPhoneUserSurvey.pdf
in camera resolution and memory capacity (Morris, 2005).

It is difficult to define the product market for an MFP from a strict functionality perspective. For example, some buyers purchase a multifunction office machines for the copy and printing functions, while other buyers might purchase the same device to utilize the scanning and fax functions. On the other hand, if we consider the introduction of an MFP after SFPs have already gained market acceptance, we could focus our attention on the product markets for the SFPs and how consumers in these markets would approach decision to purchase the MFP. In this paper, we are interested in examining this type of purchasing behavior at the individual consumer level. In general, we assume that consumers purchase the MFP if the product provides a positive utility. However, given the sequence of product introduction decisions, individual consumers who have already purchased an SFP would only purchase the MFP if the MFP provides a higher utility than the utility derived from the SFP.

The remaining part of this article is organized as follows. In the next section, we review the relevant literature and in Section 3, we present our model for optimally determining the pricing and aggregate quality level for the MFP. In Section 4, through a sensitivity analysis and extensive numerical examples, we present key managerial insights. Finally, conclusions and managerial implications of this research are discussed in Section 5.

2 Relevant Literature

Since our focus is on integrating the “quality” level of SFPs and the MFP, an obvious stream of relevant literature is on vertical differentiation. In general, vertical differentiation models assume that the manufacturer in a product market offers multiple variants that differ in quality and the heterogeneous consumers purchase a product with the highest utility. Mussa and Rosen (1978), Gabszewics and Tisse (1979), Moorthy (1984), Desai (2001) assume that the quality level has only one dimension while Baumol (1967), Vandenbosch and Weinberg (1995), and Kim and Chhajed (2002) investigate product positioning and design problem in multiple quality attributes. From the consumer perspective, heterogeneity is assumed based on the income budget distribution (Baumol, 1967; Gabszewicz and Thisse, 1979; Waterson, 1989) or on the quality preference distribution (Vandenbosch and Weinberg, 1995; Wauthy, 1996).

Mussa and Rosen’s (1978) paper is the first assuming consumers differ in their intensity of quality preference. They investigate the quality design in product line for a monopolist in a
consumer self-selection market. Moorthy (1984) incorporates the impact of competition into a vertical differentiation model by focusing on two aspects: cannibalization in variants of a product line and competition across firms. Desai (2001) discusses quality segmentation and the market coverage in spatial markets and finds that the cannibalization may not influence the price and the quality decision when the market is not fully covered.

Baumol (1967) analyzes the optimal positioning of a new product or a new store along two dimensions of product characteristics or retailer stances. The results show that the new product (or store) must exist above the boundary of a convex region formed by current products (or stores). Vandenbosch and Weinberg (1995) analyze two-dimensional vertical differentiation in a two-firm situation. The consumers have different preferences on both dimensions and they are assumed to be uniformly distributed over the population. Kim and Chhajed (2002) investigate the product line design problem with multiple quality attributes and show that: (a) when the consumer’s preferences have different orderings on different attributes, no product has a better quality in all dimensions; and (b) offering one product for all customers is never optimal.

Krishnan and Gupta (2001) model a common design platform’s impacts on product planning decisions. When market diversity or non-platform scale economies are large, sharing a common platform is not appropriate. The sharing platform also has a significant influence on product positioning and introduction sequence. Krishnan and Zhu (2006) focus on product family design of development intensive products (DIPs). Assuming two dimensions of quality, the optimal quality levels for high and low valuation segments are related to the market size. A subsumed low-end product designed by downgrading a high-end product might not be appropriate for DIPs. The authors propose an overlapped product design approach for DIPs when two quality dimensions are coupled.

Weber (2008) develops a two-stage model for information products with vertical and horizontal differentiation. The firm offers its top-quality flagship products as the first stage and extend its products by versioning the flagship product. If the marginal development cost is low, he finds that such a strategy is optimal. Sholl, Manthey, Helm & Steiner (2005) compare analytical hierarchy process and conjoint analysis in solving a multi-attribute design problem. The focus is on elicitation of respondents’ preferences and in general they find that both methodologies are equivalent with the analytical hierarchy process being a better choice for certain special cases.

Most vertical integration models assume that customer preferences are uniformly dis-
tributed (e.g., Moorthy, 1984; Vandenbosch and Weinberg, 1995; Wauthy, 1996). Ansari, Economides & Ghosh (1994) relax the quality preference distribution to a generalized beta distribution, which yields substantially different results from those assuming uniform distribution. Two papers investigate vertical differentiation over time under models of intertemporal competition (Moorthy and Png, 1982; Deneckere and de Palma, 1998). Rather than review this extensive body of literature in further detail, we refer the reader to Kaul and Rao (1995).

Two recent papers which analyze the product portfolio decision for an MFP are Chen, Vakharia, and Alptekinoglu (2008), and Chen, Carrillo, Vakharia, and Sin (2010). In both papers, the focus is on the product portfolio decision assuming that aggregate market demand incorporates substitution effects between SFP’s and an MFP. In the first paper, using a stylized model for two SFP’s and a single MFP, the authors derive the parametric conditions which are a function of substitution effects and profit margins) under which the MFP will be included in the product portfolio for the firm. In the second paper, the analysis is extended for the general case of \( n \) SFPs and a correspondingly large number of possible MFPs with a view to identifying the functionalities that should be integrated into designing MFPs and the mix of SFPs and MFPs which should comprise the optimal product portfolio. In this paper, our focus is quite different on several dimensions. First, we assume a timeline of product introductions characterized through industry practice. Thus, we analyze the MFP product introduction decisions assuming that SFPs are already in the market. Second, in modeling demand, we incorporate consumer preferences in the quality levels designed into the MFP and the SFPs. Third, we provide guidelines on the simultaneous pricing and quality level decisions for the MFP. Finally, we characterize the purchasing behavior of different customer segments when the MFP is introduced. We now proceed to describe the analytical approach in the next section.

3 Modeling Framework

3.1 Current Setting

Our stylized setting is as follows. A single firm currently markets two products \((s = 1, 2)\). Each product \( s \) is offered to individual consumers with a predetermined quality level vector \((r_{s1}, r_{s2})\) where \( r_{s1} \) \( (r_{s2}) \) is the quality level associated with functionality \( F_1 \) \( (F_2) \). An indi-
individual consumer obtains a net utility from purchasing product \( s \) (i.e., \( u_s \)) which is defined as
\[
u_s = \theta_s (\sum_{j=1}^{2} r_{sj}) - p_s\]
where \( \theta_s \) defines the consumer type in product \( s \); \( r_{sj} \) is as defined before; and \( p_s \) is the market price for product \( s \). In line with prior research (e.g., Moorthy and Png, 1992; Kim and Chhajed, 2002), this utility function incorporates the idea that consumers prefer higher quality levels as compared to lower quality levels for each functionality. Consumer types (defined by \( \theta_s \)) are heterogeneous and assumed to be uniformly spread over the interval \([0, 1]\). Given this setting, an individual consumer would purchase product \( s \) provided
\[
\theta_s > \frac{p_s}{\sum_{j=1}^{2} r_{sj}}
\]
since he/she would obtain a positive utility through the purchase. Assuming a standardized market size of 1, this indicates that the total number of consumers who would buy product \( s \) is
\[
1 - \frac{p_s}{\sum_{j=1}^{2} r_{sj}}.
\]
Since there is no loss of generality in obtaining the results, we assume a zero marginal production cost for each product (Bonanno, 1986; Choi and Shin, 1992).

To start with, each product offered by the firm incorporates a single functionality and thus, the predefined quality level vector for product 1 is \((r_{11}, 0)\) and for product 2 is \((0, r_{22})\). Thus, the firm’s profit maximization problem is as follows:
\[
\text{Maximize } p_1, p_2 \geq 0 \Pi = (1 - \frac{p_1}{r_{11}})p_1 + (1 - \frac{p_2}{r_{22}})p_2
\]
Since \( \frac{\partial^2 \Pi}{\partial p_s^2} = -\frac{2}{r_{ss}} < 0 \) for \( s = 1, 2 \), and \( \frac{\partial^2 \Pi}{\partial p_1 \partial p_2} = 0 \), it is obvious that \( \Pi \) is strictly concave in both \( p_1 \) and \( p_2 \) and thus, by setting the first order conditions (FOCs) equal to 0, we can obtain the following optimal prices:
\[
P_1^* = \frac{r_{11}}{2} \quad \text{(2)}
\]
\[
P_2^* = \frac{r_{22}}{2} \quad \text{(3)}
\]
Based on these prices, the corresponding quantities purchased are \( q_s = \frac{1}{2} \) for \( s = 1, 2 \) and the resulting optimal total firm profit is \( \Pi^* = \frac{r_{11} + r_{22}}{4} \).

3.2 Multi-function Product Introduction

At a later point in time, technological advances make it possible for the firm to introduce a multi-function product (indexed as \( s = 3 \)) with the associated quality vector \((r_{31}, r_{32})\). Both \( r_{31} \) and \( r_{32} \) are positive “quality” levels since product 3 provides multiple functionalities. Since we assume that the functional technology for integration into the multi-function product is known, we also define \( \hat{r}_{31} \) and \( \hat{r}_{32} \) as the maximum quality level for each functionality.
that can be integrated into the new multi-function product. Define \( l = r_{31} + r_{32} > 0 \) as the additive “compound” quality level (a firm level decision) and \( \hat{l} = \hat{r}_{31} + \hat{r}_{32} \) as the maximum compound quality level associated with product 3 such that \( l \leq \hat{l} \).

The potential market for the multi-function product is characterized by set of consumer types \( \theta_3 \) who are also heterogeneous and uniformly distributed in the interval \([0, 1]\). As before, an individual consumer obtains a net utility from purchasing product 3 (i.e., \( u_3 \)) which is defined as \( u_3 = \theta_3(\sum_{j=1}^{2} r_{3j}) - p_3 \). The total market for the new multi-function product can be conceptualized as consisting of the following segments:

- **Segment 1:** consists of those customers who previously purchased product 1 (i.e., \( u_1 > 0 \)) and did not purchase product 2. These customers would purchase multi-function product 3 only if it would provide a greater utility than that provided by product 1, i.e., \( u_3 > u_1 \). Let the customers in this segment be equal to \( q_{31} \).

- **Segment 2:** consists of those customers who previously purchased product 2 (i.e., \( u_2 > 0 \)) and did not purchase product 1. These customers would purchase multi-function product 3 only if it would provide a greater utility than that provided by product 2, i.e., \( u_3 > u_2 \). Let the customers in this segment be equal to \( q_{32} \).

- **Segment 3:** consists of those customers who previously did not purchase either product 1 or product 2. These customers would purchase the multi-function product 3 only if it provided them with a net positive utility, \( u_3 > 0 \). Let the customers in this segment be equal to \( q_{33} \).

- **Segment 4:** consists of those customers who previously purchased products 1 and 2 (i.e., \( u_1 > 0 \) and \( u_2 > 0 \)). These customers would purchase multi-function product 3 only if it would provide a greater utility than that provided by either product 1 or product 2. Analytically, this implies that \( u_3 > \max(u_1, u_2) \). Let the customers in this segment be equal to \( q_{34} \).

Based on this, the aggregate total market for the multi-function product 3 is defined as \( q_3 = q_{31} + q_{32} + q_{33} + q_{34} \). The analytical computations for determining each of the market segments are detailed in Appendix A and based on this the aggregate total market for multi-function product 3 (i.e., \( q_3 \)) is:

\[
q_3 = q_{31} + q_{32} + q_{33} + q_{34}
\]
\[ k = r_{11}^2 + r_{22}^2 + 6r_{11}r_{22}; \quad n = r_{11}r_{22}; \quad \text{and} \quad m = (r_{11} + r_{22})^3. \]

For the MFP product introduction cost, we assume that this cost is a function of the compound quality level and depends on how well two functions in MFP 3 can be integrated and consolidated. Thus, we assume that the quality cost for this scenario is convex and increasing in the compound quality level. More specifically, the costs are assumed to be \( hl^3 \), where \( h > 0 \) represents the multi-function technology integration parameter. In line with the assumption in Mussa and Rosen (1978), this cubic parametrization of costs as a function of the compound quality implies that the unit and marginal costs are increasing functions of quality, i.e., \( \frac{dl^3}{dp} = 3hl^2 > 0; \) and \( \frac{d^2l^3}{dp^2} = 6hl > 0 \) for all \( l \geq 0. \)

The firm’s problem is to simultaneously determine the market price \( (p_3) \) and compound quality level \( (l) \) of MFP 3. This problem from a profit maximization perspective is as follows:

Maximize \( p_3 \geq 0; 0 \leq l \leq \hat{l} \) \( \Pi = q_3p_3 - hl^3 \)

\[ = [(1 - \frac{p_3}{l}) \frac{k}{8n} - \frac{m}{48ln}]p_3 - hl^3 \]  

where \( k = r_{11}^2 + r_{22}^2 + 6r_{11}r_{22}; \quad n = r_{11}r_{22}; \) and \( m = (r_{11} + r_{22})^3. \)

The first- and second- order conditions of \( \Pi \) with respect to \( p_3 \) and \( l \) are:

\[ \frac{\partial \Pi}{\partial p_3} = \frac{6k(l - 2p_3) - m}{48ln} \]  

\[ \frac{\partial \Pi}{\partial l} = \frac{p_3(m + 6kp_3)}{48l^2n} - 3hl^2 \]  

\[ \frac{\partial^2 \Pi}{\partial p_3^2} = \frac{-k}{4ln} \]  

\[ \frac{\partial^2 \Pi}{\partial l^2} = \frac{-p_3(m + 6kp_3)}{24l^3n} - 6hl \]  

\[ \frac{\partial^2 \Pi}{\partial p_3 \partial l} = \frac{m + 12kp_3}{48l^2n} \]  

Assuming an interior solution (i.e., \( p_3 > 0 \) and \( 0 < l < \hat{l} \)), the second-order conditions in equations (8) and (9) are < 0 and hence, to show that this problem is strictly and jointly concave in \( p_3 \) and \( l \), we need to ensure that:

\[ |H_2| = \left( \frac{\partial^2 \Pi}{\partial p_3^2} \right) \left( \frac{\partial^2 \Pi}{\partial l^2} \right) - \left( \frac{\partial^2 \Pi}{\partial p_3 \partial l} \right)^2 \]

\[ = \frac{3kh}{2n} - \frac{m^2}{2304l^4n^2} \]  

8
is non-negative. It can be shown (see Appendix B) that for $h \geq \frac{3k^3}{8m^2n}$, $|H_2| \geq 0$ and hence, under such a parametrization of the product introduction cost parameter $h$, equation (5) is jointly concave in $p_3$ and $l$. However, after setting the first-order conditions shown in equations (6) and (7) equal to 0, it is still difficult to determine an optimal solution. Hence, we utilize the result in the Theorem below to design an efficient search algorithm which will provide an optimal solution for the firm’s profit maximization problem.

**Theorem 1**: Given any value of $l \left(\frac{m}{6k} < l \leq \hat{l}\right)$, the optimal price for the multi-function product is:

$$p_3^*(l) = \frac{l}{2} - \frac{m}{12k}$$  \hspace{1cm} (12)

**Proof**: The proof follows from the fact that for any given feasible value of $l$, equation (5) is strictly concave in $p_3$ since $\frac{\partial^2 \Pi_3}{\partial p_3^2} < 0$ (see equation (8) above). Hence, by setting the first order condition given in equation (6) equal to 0, we derive the result given in equation (12) above.

The result in this theorem is used in structuring the following proposed algorithm.

1. Set $l = \frac{m}{6k}$, price $= 0.0$, quality $= 0.0$, and $Prof = -9999999999$.
2. $l = l + 0.1$. If $l > \hat{l}$, goto 7; else goto 3.
3. Compute $p_3^*(l)$ from equation (12).
4. Compute $\Pi_3[l, p_3^*(l)] = [(1 - \frac{p_3^*(l)}{l}) \frac{k}{8n} - \frac{m}{48ln}] p_3^*(l) - hl^3$.
5. If $\Pi_3[l, p_3^*(l)] > Prof$, set $Prof = \Pi_3[l, p_3^*(l)]$, price $= p_3^*(l)$, and quality $= l$.
7. Set $p_3^* = price$; $l^* = quality$, and $\Pi_3^* = Prof$. Stop.

Based on the Theorem above, this algorithm is guaranteed to converge to a global optimal solution even if equation (5) is not strictly and jointly concave in the decision variables. On the other hand, structural insights into the impact of key parameters obviously cannot be obtained. Thus, we describe a set of extensive numerical experiments in the next section which allow us to provide key insights into the optimal pricing and quality strategies for the multi-function product.
4 Numerical Analysis

In this section, we use several numerical examples to illustrate the firm’s optimal multifunction product pricing and quality strategies. We choose Canon as the manufacturer that offers scanners (\(s = 1\)) and printers (\(s = 2\)) and a multi-function printer-scanner (\(s = 3\)). Since we are not able to directly obtain estimates of the initial quality levels for the two single function products (i.e., \(r_{11}\) and \(r_{22}\)), we first note that the average prices (listed on Nextag.com) are $62.5 (i.e., \(p_1 = 62.5\)) and $100 (i.e., \(p_2 = 100\)). Under the assumption that the firm has optimally priced these two single function products, this implies that the imputed quality level \(r_{11} = 2 \times p_1 = 125\) and \(r_{22} = 2 \times p_2 = 200\). We assume the upper bound of MFP 3’s quality level is at 600 (i.e., \(\hat{l} = 600\)) based on current best fusion technology.

To incorporate other possible scenario, below, we also investigate other situations that two SFPs have a very large or very small gap in their quality levels (\(RV\ gap\) thereafter).

Based on these observed market prices, we consider three basic scenarios:

- **Scenario I:** For this case, we set \(r_{11} = r_{22} = 125\). This scenario reflects a setting where the inherent “quality” of the second product is “equal” to that of the first product (RV gap is zero). In this case, the optimal prices for the single function products are: \(p_1^* = p_2^* = 62.5\). Given a market size of 1000 customers, this translates to each single function product again capturing 500 customers and based on this the firm profits are $62,500.

- **Scenario II:** For this case, we set \(r_{11} = 125\) and \(r_{22} = 200\). This scenario reflects the current setting which has a small RV gap since these quality levels results in the optimal prices for the two SFPs as: \(p_1^* = 62.5\), and \(p_2^* = 100\). Given a market size of 1000 customers, this translates to each single function product capturing 500 customers and based on this the firm profits are $81,250.

- **Scenario III:** For this case, we set \(r_{11} = 125\) and \(r_{22} = 380\). This scenario reflects a setting where the inherent “quality” of the second product is substantially higher than that of the second product (RV gap is large). In this case, the optimal prices for the single function products are: \(p_1^* = 62.5\), and \(p_2^* = 190\). Given a market size of 1000 customers, this translates to each single function product again capturing 500 customers and based on this the firm profits are $126,250.
In addition, we assume a market size of 1000 customers for illustration purposes. Due to that scaling, in all our experiments, we vary the value of \( h \) from 0.0002 to 0.001 with an increment of 0.0002 to understand the impact of fusion design difficulty. The optimal price and quality levels of the MFP for each of these settings for each scenario are shown in Table 1.

| Insert Table 1 about here |

When the fusion technology difficulty is low \((h = 0.0002)\), we find that all three scenarios should design the MFP to its maximum, which is the upper bound of MFP’s quality level, \( \hat{l} = 600 \). As the fusion technology difficulty \((h)\) increases, we find the optimal quality level of MFP 3 \((l^*)\) decreases in all scenarios. When other parameters remain the same, the optimal quality level of MFP 3 in scenario III is higher than that in scenario II, which is again higher than that in scenario I. This is partially because SFP 2’s RV in scenario III is higher than that in scenario II, which is again higher than that in scenario I. However, the difference of optimal quality level of MFP 3 in three scenarios is in a much less magnitude compared to the difference of RV of SFP 2 in three scenarios. The derived influence of the optimal quality level of MFP 3 \((l^*)\) on the optimal price of MFP 3 \((p^*_3)\) is even more complex. From the optimal solution provided Theorem 1, we know the optimal price of MFP 3 is a function of MFP 3’s quality level, which includes a portion of price discount compared to the optimal price of SFPs (see (2), (3), and (12)). At the same level of fusion technology difficulty, there is no direct relationship between the optimal quality level and the optimal price of MFP 3. However, when we use \( p^*_3/l^* \) for each cases we can find that \( p^*_3/l^* \) becomes smaller when (i) the fusion technology difficulty becomes larger (larger \( h \)); (ii) the RV gap between two SFPs increase (e.g., Scenario III). When \( h = 0.0002 \) in scenario I, the firm should offer a 3.4% of price discount \(((50\% - 48.3\%)/50\%)\) for MFP 3 compared to the optimal price of SFPs \((p^*_3/l^* = 50\%)\) when MFP 3 is not available. But when \( h = 0.001 \) in scenario III, the firm should offer a 15.6% of price discount \(((50\% - 42.2\%)/50\%)\) for MFP 3. In a managerial interpretation, it means that the firm should offer more price discount on the MFP when the fusion technology difficulty and the RV gap between two SFPs increase.

Based on the magnitude of MFP 3’s optimal quality level \((l^*)\) in relative to \( r_{11} + r_{22} \), we can define three relationships: (i) synergistic fusion \((l^* > r_{11} + r_{22})\): the firm should seek synergy while fusing two functions together such that the consumers are willing to pay more for the MFP; (ii) additive fusion \((l^* = r_{11} + r_{22})\): the firm should maintain the quality level
(as well as the willingness-to-pay from the consumers) while fusing two functions together; and, (iii) diminishing fusion \( l^* < r_{11} + r_{22} \): the firm should avoid over-investment on the design of MFP 3 (that is, the optimal quality level of MFP 3 should be lower than lower than the sum of \( r_{11} \) and \( r_{22} \)). Based on the results in Table 1, a synergistic fusion strategy is favored when the RV gap is small (scenario I) and a diminished fusion strategy is favored when the RV gap is large (scenario III).

We also notice that a small value of \( h \) (low fusion technology difficulty) favors synergistic fusion strategy and a larger of \( h \) favors diminishing fusion strategy. Comparing three scenarios, the firm in scenario I is more likely to seek synergy and design a better quality of MFP (especially when the fusion technology difficulty is low). The firm in scenario III should avoid over-design the MFP especially when the fusion technology difficulty is high. The reason of this phenomenon is because different firms should focus on different segments of MFP potential customers (which will be explained later). Note that the additive fusion strategy is only shown scenario II when \( h = 0.0008 \) in Table 1 because the additive fusion strategy is the border between the synergistic fusion and the diminishing fusion strategy. In our three scenarios, additive fusion strategy for scenarios I, II and III to be optimal when \( h \) is 1.32x10^{-3}, 8.1x10^{-4}, and 3.8x10^{-4}, respectively. As a result, as the RV gap increases, the threshold value of \( h \) to render an optimal additive fusion strategy decreases.

One interesting result in Table 1 is that, compared to scenarios I & II, scenario III always renders a higher optimal quality level of MFP 3, but not necessary a higher optimal price of MFP 3. In the case that \( h \) is too small (e.g., \( h = 0.0002 \)) such that \( l^* \) is set at its maximal value of \( \hat{l} \), the relationship between \( l \) and \( p_3 \) is a special case with bordered value of \( \hat{l} \). When \( h \) is large enough and \( l^* \) is not set as \( \hat{l} \), the relationship between \( l^* \) and \( p_3^* \) is not a simple correlation either. When \( h = 0.0004 \) a larger \( l^* \) results in a larger \( p_3^* \) for all three scenarios. However, when \( h = 0.001 \), scenario III (which the RV gap is the largest) should set the quality level of MFP 3 higher than scenarios I and II, but its optimal price of MFP 3 should be set lower than scenarios I and II. Here, it demonstrates again that when the RV gap is large, the firm should offer a relatively larger price discount to attract more SFP users to upgrade to the MFP.

One strength of our model is that it not only provides the optimal quality and price of the MFP, it can also determines the numbers of consumers coming from four segments \( (q_{31}, q_{32}, q_{33}, \text{and } q_{34}) \), which are discussed in section 3.2. When two SFPs have a zero RV gap as in scenario I, the numbers of consumers from segments 1 & 2 are equal. But when two SFPs
have a large RV gap, the number of consumers from segment 1 (who bought SFP 1 only) become larger than the number of consumers from segment 2 (who bought SFP 2 only) when MFP 3’s price set as $p^*_3$. This is because the buyers of SFP 2 has a higher utility surplus from buying SFP 2, hence, it forms a higher threshold to justify buying MFP 3. Note that the magnitude difference between $q_{31}$ and $q_{32}$ for scenarios II and III becomes larger as the fusion technology difficulty ($h$) increases.

The results of $q_{33}$ and $q_{34}$ in Table 1 show different patterns. Overall, the value of $q_{33}$ in various conditions is less variant compared to $q_{32}$ and $q_{34}$. This is because the buyers from segment 3 (which bought neither SFP 1 nor SFP 2) bought none of SFP 1 or SFP 2 and will choose to buy MFP 3 when MFP 3 provides a positive utility. With MFP 3’s price sets as $p^*_3$, in scenario I, there are always more consumers from segment 3 than that from segment 4 (which bought SFPs 1 & 2); however, the result is the opposite in scenario III, which MFP 3 has more consumers from segment 4 than from segment 3. Because we assume that the consumers in segment 4 would purchase MFP 3 as long as the utility surplus from MFP 3 is higher than the lower utility surplus of SFPs 1 & 2, it is not surprising to see that a higher RV gap can attract more buyers from segment 4 when the firm sets at the optimal price and provides a deeper price discount for MFP 3.

5 Conclusion and Future Research

MFPs have gradually merged several markets that were served by distinct SFPs before into one market. As the manufacturers of SFPs seek for continuous revenue stream, they design MFPs that integrate several functions to (i) attract those consumers who have bought SFPs to ‘upgrade’ and (ii) deliver the message to those consumers who has not bought any SFPs that “it is time to purchase.” We propose a model that the consumer’s purchase decision is based on individual utility functions and we are interested in the optimal quality and pricing decisions for the manufacturer to offer an MFP after two SFPs have been offered in the first stage. Different from previous models that analyzes from aggregate demand (Chen et al., 2008, 2010), we not only solve the optimal pricing and quality decisions for the MFP and derive the quantity to offer, but also identify four customer segments of MFP’s demand and derive how many consumers actually come from each segments. Finally, we define three fusion strategies – diminishing, additive and synergistic – and find the conditions that favors each one of them.
We also obtain several managerial insights from our results. First, when offering the MFP after two SFPs have been offered, the firm needs to provide “upgrade discount” in the price of the MFP to optimize its profit. If the firm sets MFP’s price following the formula of its SFP’s optimal prices (e.g., (2) and (3)), then the firm eventually gains less consumers and hurts its profit. The numerical results quantify the price discount for each scenario. When the RV gap between two SFPs is larger and/or the fusion technology difficulty become larger, the firm should offer a higher price discount.

Second, by following our optimal strategy, the firm will gain more customers from the segment that its consumers only buy the SFP with a relatively low RV (in our numerical examples, it is SFP 1). The difference between the potential buyers for those who only buy low RV SFP and for those who only buy high RV SFP becomes more significant when the RV gap between two SFPs becomes larger. For the segment that its consumers buy none of SFPs, the impact of the RV gap between SFPs is not as important as for the segment that its customers buy both SFPs.

Third, when the firm optimizes the price and the quality level of the MFP, the diminishing fusion strategy is favored when the fusion technology parameter is high and when the RV gap is large. The synergistic fusion strategy is optimal when the fusion technology parameter is low and the RV gap is low. The border where diminishing and synergistic fusion strategies meet is the situations that additive fusion strategy is optimal.

There are several directions exploring the diffusion of MFPs that warrant further research. First, an empirical research that examines the fitness of our proposed model is highly encouraged. In the markets of office machines, cellular phones, home entertainment electronics, there exist many MFPs that cover the needs of SFPs in various scenarios. For example, the printer, the scanner and the all-in-one office machines fit into scenarios I and IV; the TV, the DVD player, and the TV-DVD player combo fit into scenarios II and IV; and the cellular phone and the digital music player may fit into scenario III and VI. It will be of business’ interest to know whether the fusion strategy proposed in our model really reflects the business practice.

Second, many SFPs still thrive after the launch of the MFP, but others become lackluster. Firms may try to rejuvenate the market for the SFPs by adding new functions, improving the quality and identifying new applications for the product. For example, after the launch of the inkjet office machine MFP, an additional market for the high end inkjet SFP helped to fuel future growth: digital photo printing. A research study that further
characterizes the properties of a thriving SFP would provide constructive guidelines to these industries.

Finally, an explicit treatment of competition on the FP decision making is warranted. From manufacturers in all SPs, which firm has a competitive advantage in offering a fusion product? The transformation of electronics product market provides many good cases which offers insights in product design and offering strategy for the manufacturers.
References


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Appendices

Appendix A

To calculate the value of $q_3 = q_{31} + q_{32} + q_{33} + q_{34}$, we use an indirect approach. We introduce four interim variables: $z_{31}$ contains the consumers whose $u_3 > u_1 > 0$, $z_{32}$ contains the consumers whose $u_3 > u_2 > 0$, $z_{33}$ contains the consumers whose $u_3 > 0 \cap u_1 = 0 \cap u_2 = 0$, and $z_{34}$ contains the consumers whose $u_3 > u_1 > 0 \cap u_3 > u_2 > 0$. It is clear that $q_3 = z_{31} + z_{32} + z_{33} - z_{34}$.

For $z_{31}$, the consumers must have $u_3 > u_1$ such that they are willing to upgrade from SFP 1 to MFP 3. Because $u_3 = \theta_3l - p_3$ and $u_1 = \theta_1r_{11} - p_1 = \theta_1r_{11} - 0.5r_{11}$, for the consumer type to upgrade from SFP 1 to MFP 3, the condition $\theta_3 > \frac{p_3+(\theta_1-0.5)r_{11}}{l}$ must hold. Similarly, for the consumer type to upgrade from SFP 2 to MFP 3, the condition $\theta_3 > \frac{p_3+(\theta_2-0.5)r_{22}}{l}$ must hold. $z_{33}$ contains the consumers who buys none of SFPs 1 and 2 and now will buy MFP 3 if $u_3 = \theta_3l - p_3 > 0$. In our definition, $z_{34}$ represents the overlap of $z_{31}$ and $z_{32}$; as a result, the consumer type in $z_{34}$ must have $\theta_3 > \max\{\frac{p_3+(\theta_1-0.5)r_{11}}{l}, \frac{p_3+(\theta_2-0.5)r_{22}}{l}\}$.

If the left term is larger, i.e. $u_1 > u_2$, then $\theta_1 > (\theta_2 - 0.5)\frac{r_{22}}{r_{11}} + 0.5$ must hold. If the right term is larger, i.e. $u_1 < u_2$, then $\theta_2 > (\theta_1 - 0.5)\frac{r_{22}}{r_{11}} + 0.5$ must hold. Now we can calculate $z_{31}$, $z_{32}$, $z_{33}$, $z_{34}$ and $q_3$.

\[
\begin{align*}
z_{31} &= \int_{0.5}^{1} \int_{0}^{1} \int_{\frac{p_3+(\theta_1-0.5)r_{11}}{l}}^{1} 1 d\theta_3 d\theta_1 = \frac{1}{2} \left(1 - \frac{p_3}{l}\right) - \frac{r_{11}}{8l} \\
z_{32} &= \int_{0.5}^{1} \int_{0}^{1} \int_{\frac{p_3+(\theta_2-0.5)r_{22}}{l}}^{1} 1 d\theta_3 d\theta_2 = \frac{1}{2} \left(1 - \frac{p_3}{l}\right) - \frac{r_{22}}{8l} \\
z_{33} &= \int_{0}^{0.5} \int_{0}^{1} \int_{\frac{p_3+(\theta_1-0.5)r_{11}}{l}}^{1} 1 d\theta_3 d\theta_2 d\theta_1 = \frac{1}{4} \left(1 - \frac{p_3}{l}\right) \\
z_{34} &= \int_{0}^{0.5} \int_{\frac{\theta_2-0.5)r_{22}}{r_{11}}+0.5}^{1} \int_{\frac{p_3+(\theta_1-0.5)r_{11}}{l}}^{1} 1 d\theta_3 d\theta_1 d\theta_2 + \int_{0}^{0.5} \int_{\frac{\theta_2-0.5)r_{22}}{r_{11}}+0.5}^{1} \int_{\frac{p_3+(\theta_2-0.5)r_{22}}{l}}^{1} 1 d\theta_3 d\theta_2 d\theta_1 \\
q_3 &= z_{31} + z_{32} + z_{33} - z_{34}
\end{align*}
\]

\[
\begin{align*}
q_3 &= \frac{3}{4} \left(1 - \frac{p_3}{l}\right) + \frac{1}{8} \left(\frac{r_{11}}{r_{22}} + \frac{r_{22}}{r_{11}}\right) \left(1 - \frac{p_3}{l}\right) - \frac{r_{11} + r_{22}}{16l} - \frac{1}{48l} \left(\frac{r_{11}^2}{r_{22}} + \frac{r_{22}^2}{r_{11}}\right)
\end{align*}
\]
\[\begin{align*}
&= (1 - \frac{p_3}{l}) \frac{r_{11}^2 + r_{22}^2 + 6r_{11}r_{22}}{8r_{11}r_{22}} - \frac{r_{11}^3 + r_{22}^3 + 3r_{11}^2 r_{22} + 3r_{11}r_{22}^2}{48l r_{11}r_{22}} \\
&= (1 - \frac{p_3}{l}) \frac{r_{11}^2 + r_{22}^2 + 6r_{11}r_{22}}{8r_{11}r_{22}} - \frac{(r_{11} + r_{22})^3}{48l r_{11} r_{22}}
\end{align*}\]

Set \( k = r_{11}^2 + r_{22}^2 + 6r_{11}r_{22} \), \( m = r_{11} + r_{22} \) and \( n = r_{11}r_{22} \) and hence, \( q_3 = (1 - \frac{m}{l}) \frac{k}{8n} - \frac{m}{48ln} \).
Appendix B

We know that:

$$|H_2| = \left(\frac{\partial^2 \Pi}{\partial p_3^2}\right)\left(\frac{\partial^2 \Pi}{\partial l^2}\right) - \left\{\left(\frac{\partial^2 \Pi}{\partial p_3 \partial l}\right)\right\}^2$$

$$= \frac{k p_3 (m + 6 k p_3)}{96 l^4 n^2} + \frac{3 k h}{2 n} - \frac{m^2 + 24 m k p_3 + 144 k^2 p_3^2}{2304 l^4 n^2}$$

$$= \frac{3 k h}{2 n} - \frac{m^2}{2304 l^4 n^2}$$

To show that $|H_2| \geq 0$, it is enough to show that:

$$\frac{3 k h}{2 n} \geq \frac{m^2}{2304 l^4 n^2}$$

$$\Rightarrow h \geq \frac{m^2}{3456 k l^4 n}$$

Now equation (12) implies that for a non-negative price, $l \geq \frac{m}{6 k}$. Substituting $l = \frac{m}{6 k}$ in the expression above we get:

$$h \geq \frac{m^2}{3456 k l^4 n}$$

$$\geq \frac{3 k^3}{8 m^2}$$
Founded in 1892, the University of Rhode Island is one of eight land, urban, and sea grant universities in the United States. The 1,200-acre rural campus is less than ten miles from Narragansett Bay and highlights its traditions of natural resource, marine and urban related research. There are over 14,000 undergraduate and graduate students enrolled in seven degree-granting colleges representing 48 states and the District of Columbia. More than 500 international students represent 59 different countries. Eighteen percent of the freshman class graduated in the top ten percent of their high school classes. The teaching and research faculty numbers over 600 and the University offers 101 undergraduate programs and 86 advanced degree programs. URI students have received Rhodes, Fulbright, Truman, Goldwater, and Udall scholarships. There are over 80,000 active alumnae.

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Our responsibility is to provide strong academic programs that instill excellence, confidence and strong leadership skills in our graduates. Our aim is to (1) promote critical and independent thinking, (2) foster personal responsibility and (3) develop students whose performance and commitment mark them as leaders contributing to the business community and society. The College will serve as a center for business scholarship, creative research and outreach activities to the citizens and institutions of the State of Rhode Island as well as the regional, national and international communities.

The creation of this working paper series has been funded by an endowment established by William A. Orme, URI College of Business Administration, Class of 1949 and former head of the General Electric Foundation. This working paper series is intended to permit faculty members to obtain feedback on research activities before the research is submitted to academic and professional journals and professional associations for presentations.

An award is presented annually for the most outstanding paper submitted.