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On Multiobjective Combinatorial Optimization and Dynamic Interim Hedging of Efficient Portfolios

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On Multiobjective Combinatorial Optimization and Dynamic Interim Hedging of Efficient Portfolios

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Abstract

This research introduces a new mixed-integer nonlinear goal program (MINLGP) with branch and bound constraints and a separable programming foundation. The motivation for creating the MINLGP algorithm is to advance the ability of portfolio managers facing multiple and hierarchical goals to simultaneously solve for an efficient portfolio with an optimal number of contingent claim contracts in order to hedge temporal portfolio volatility. Upon establishing convexity characteristics, the MINLGP is executed in a dynamic trading strategy to compare performance characteristics of a hedged optimal portfolio to those of a naively diversified portfolio. We find that a hedged equally weighted small portfolio and a hedged efficiently diversified small portfolio perform similarly when comparing risk-adjusted return metrics. However, when percentile risk measures are used to measure performance, the hedged optimally diversified portfolio clearly produces less expected catastrophic loss than that of the non-hedged and naively diversified counterpart at a predetermined confidence level.

Keywords: Nonlinear goal programming, Mixed-integer programming, Efficient Frontier, Minimum variance hedge ratio

1. Introduction

The seminal work of Markowitz (1959) introduced the one-period bi-criteria mean-variance (MV) optimization model; an approach where portfolio expected return is maximized and its risk scalar is minimized under the assumption of long run temporal stability of the sample covariance matrix. A Markowitz efficient investor is one that is assumed to have MV preferences and one that seeks to locate the highest feasible Sharpe-ratio portfolio. The benefits of MV diversification have been thoroughly interrogated by both simulation (Evans and Archer, 1968) as well by analytical derivation (Elton and Gruber, 1977). Today, there is an ongoing effort to achieve temporally high Sharpe ratios by a number of alternate schemes predicated on MV diversification objectives. These include, but are not limited to, asset weighting strategies (e.g., momentum versus contrarian strategies as reviewed by Blitz and van Vliet (2008), leveraging low beta assets (Frazzini and Pedersen, 2011), seeking alpha as a means of investing in areas that have insignificant correlation with the market (Lo, 2008), volatility hedging of idiosyncratic risk using futures contracts (Collins and Fabozzi, 1999) and trading ETFs (Michalik and Schubert, 2009). Although these different schemes are intriguing none have proven to be rationally efficient in describing how investors’ maximize expected utility. For example, a review provided by Mehrling (2005) provides a good view of contradictory evidence as it relates to beta-weighting of portfolios. He reports how over-weighting a portfolio towards high-beta assets may actually result in lower Sharpe ratios compared to leveraging low-beta assets. Similarly, Daryl and
Shawn (2012) report that the relative benefits of diversification tend to be greater for small portfolios that are equally weighted (in contrast to MV weighted) with a crossover effect as the size of the portfolio increases.

Asset diversification approaches aside, more contemporary investors have turned to high frequency (HF) or near high frequency (NHF) trading strategies to maximize expected terminal wealth from the realized spread over a holding period that is often measured in fractions of a second to minutes (Cartea and Jaimungal, 2012). In their discussion of how stochastic jumps can be caused with the arrival of HF market orders, Cartea and Jaimungal, differentiate HF trade strategies from lower frequency ones by providing a mathematical description of the HF trader and their need to trade-off expected profits against the variance and distribution of: position profits, terminal inventory and lifetime inventories. The market characteristics of HF trading are certainly interesting, but the dynamic trading environment assumed under a HF trade strategy is incongruent with the investor who seeks to hold a Markowitz efficient portfolio over discrete investment intervals.

Standing in partial alliance with portfolio managers, investment analysts, fundamentalist, and technicians – all of whom trade market instruments over longer time frames – are the applied mathematicians from the operations research and computational finance communities (subsequently referred to as ‘quants’). Quants have uniformly approached the problem of risk-adjusted performance as if it were a dynamic programming problem. That is, quants have a more expansive approach on how best to achieve higher Sharpe ratios. They suggest breaking an investment strategy into various polymorphic sub-problems so that the research can focus on crystallizing new optimization tools that are fine-tuned for the decision characteristics of a specific sub-problem.

Coincidently, quants have also helped to evolve the field of risk management by formulating the quantitative support to include the use of contingent claims to mitigate non-diversifiable risk (Collins and Fabozzi, 1999) among both small and large hedgers (Chang et al., 2000). Today, contingent claim analysis plays a crucial role in achieving optimal solutions to portfolio planning problems by including derivative securities. With these component concepts in place, we can state the intended contribution of this paper.

The primary objective of this paper is to advance normative portfolio decision-making by coalescing optimization algorithms in combinatorial-, nonlinear- and multiple objective-programming to produce a new mixed-integer nonlinear goal programming algorithm (MINLGP). Although the new MINLGP is designed to have a wide domain of applicability, the specific interest of this paper is directed towards advancing normative solutions to more realistic models of investor diversification behavior. To that end, we employ the MINLGP algorithm to solve a multiple objective bi-criteria MV diversification model to include an “if-then” binary variable to simultaneously determine hedging. When hedging is indicated the solution will identify the non-fractional value of stock-index futures to sell. Multiple objective combinatorial optimization design problems are known to pose substantial computational difficulty (Ehrgott and Gandibleux, 2002). Knowing this fact, we enhance the computational effectiveness of the
new MINLGP algorithm by relying upon the computationally less demanding off-diagonal Sharpe portfolio model (Sharpe MINLGP) to generate an efficient frontier (Sharpe, 1971). The choice of portfolio model notwithstanding, we note that with additional constraints and control variables, the MINLGP algorithm presented in this research is capable of solving the full, and computationally more complex, covariance model of Markowitz.

The remainder of the paper is organized as follows. Section 2 presents the statement of the problem. This is followed by section 3 where we provide a theory for the temporal volatility hedging of multi-objective efficient portfolios. Section 4 is devoted to a normative experiment wherein we also examine what, if any, loss of generality occurs by choosing the Sharpe diagonal model within a multi-objective framework. Lastly, as part of the normative exercise, we address temporal performance metrics for sample portfolios. In section 5 we provide a summary and conclusion.

2. **Problem Definition**

Investors diversify and/or hedge investment portfolios in order to mitigate the financial risk of not achieving a desired rate of return. Owing to a myriad of economic factors, particularly the substantial volatility in interest rates and exchange rates that dates to the 1970s and 1980s, evidence mounted that MV diversification alone does not mitigate all portfolio risk. In fact, recent evidence continues to suggest that investors now understand that a portfolio’s market risk is best mitigated by hedging with contingent claims (Arsic, 2005). With findings tracing back to the introduction of the Value Line Index futures contract, Sharda and Musser (1986) were at the forefront when they introduced a discrete multi-period goal programming model to invoke a static futures-based hedge ratio to mitigate the risk of an investment portfolio. Subsequent research extended single period futures hedging to include reference securities that may or may not be owned by the parties involved. By way of example, Brooks et al. (2006) provide evidence of efficient cross hedges using U.S. based single-stock futures. While investors all agree that hedging a portfolio with stock index futures is sound risk mitigation, the question remains as to how this should be done.

2.1 **Which Hedge Ratio?**

Many of the early volatility hedging investigations focused on the specification of static regression-based hedge ratios. More contemporary studies focused on the time-varying properties under the GARCH framework (for a review and global implications see, McMillan and Garcia (2010)). As an alternative, Hsu et al. (2008) proposed a class of new copula-based GARCH models to estimate risk minimizing dynamic time-varying hedge ratios. These authors reasoned that copula methods are statistically designed to take into account the changing joint distribution of spot and futures returns. The study compared the constant correlation GARCH of Bollerslev (1990), the multivariate GARCH BEKK model of Engle and Kroner (1995), the dynamic conditional correlation (DCC) GARCH model proposed by Engle and Sheppard (2001), and the traditional approach proffered by Peters (2008). The reported results found that the
traditional (static) regression-based hedge ratios are inefficient when employed over long time-intervals. Lee (2009) extended GARCH based estimation to include a regime switching copula model to account for time-varying hedge ratios. The contributions these studies make in their respective examinations of short- and long-term hedging provide a direct contribution to the purpose behind the development of the temporal MINLGP. Specifically, because the duration of any specific hedge in the proposed normative application is not over a long period and because the data used in the study is not HF transaction data with captured jumps this study defaults to the static and well known minimum variance hedge ratio (MVHR) of Ederington (1979) and Johnson (1960).

2.2 Which Goal Programming Methodology?

In the goal programming approach to decision analysis it is not always possible to satisfy all objectives simultaneously. Society’s need to model increasingly complex decision problems has promoted the development and application of linear- and combinatorial-goal programming (LGP and MILGP) to allocation problems in the medical sciences (Trivedi, 1981), energy management (Mavrotas and Diakoulaki, 2005), and job-shop scheduling (Lashine et al., 1991; Ozguven et al., 2012). Similarly, advances in convex nonlinear multiple objective modeling have also surfaced in the literature. The relevance of convexity properties is discussed at length in the literature from stochastic analysis to financial optimization to mathematical finance (for a review, see Pennanen (2012)). As investors seek portfolio diversification strategies to maximize MV expected utility, their decisions are naturally modeled as a convex optimization problem (Ruszczyński and Shapiro, 2004). By way of example, Dash and Kajiji (2002) derived a solution to the stochastic bank ALM optimization model with nonlinear demand and supply functions. Subsequently, Dash and Kajiji (2005) viewed the management of a property-liability insurer as a twin set of efficient portfolios divided only by the accounting identity. The Dash and Kajiji research modeled the complex portfolio problem using the NLGP method. What the section review provides is evidence that optimization disciplines routinely obtain efficient solutions to large-scale decision problems by dividing the problem into smaller components that are more amenable to recognized and targeted optimization tools. The MINLGP algorithm is a significant advance as it brings together multiple optimization concepts while permitting alternate optimization sub-problems to stand on their own.

3. Multiple Objective Volatility Hedging of MV Portfolios

In this section the paper presents the mathematical underpinnings of the new algorithm. The exposition is made within the context of our purpose – to dynamically hedge efficient portfolios. Section 3 proceeds as follows. First, we review the importance of the convexity assumption in financial modeling. This is followed by a section that establishes the ability of convex goal programming to solve MV-theoretic models. A proof that the goal programming algorithm can be advanced to include the combinatorial if-then constraint is subsequently added to the analysis. Lastly, we review how separable programming methods are useful for producing a convex
linearization of the MINLGP augmented with branch-and-bound constraints (see Chinneck (2012), for a discussion of the classic branch and bound method).

### 3.1 Multi-Objective Integrability for Temporal Interim Period Trading

We paraphrase Czichowsky and Schweizer (2012) who describe a semimartingale financial market with general convex constraints on trading strategies. Under their construction, we define $S = (S_t)_{0 \leq t \leq T}$ to be an $\mathbb{R}^n$-valued semimartingale model of the discounted prices of $n$ risky assets. The initial trading strategy is constructed as a non-levered and non-hedged commitment of wealth. The initial wealth of this investment strategy is described by $w \in \mathbb{R}$ and a $\mathbb{R}^n$-valued predictable process $\vartheta = (\vartheta_t)_{0 \leq t \leq T}$ which describes the dynamic share weighting scheme over time. The resulting final wealth is:

$$W_T(w, \vartheta) := w + \int_0^T \vartheta_t dS_t =: w + G_T(\vartheta)$$

Now if the $\mathcal{F}_T$-measurable random variable $H$ gives the time $T$ payoff of a financial product, then the MV hedging for $H$ is to solve the (linear-quadratic control) problem

$$\text{Min} : E \left[ |H - w - G_T(\vartheta)|^2 \right],$$

either over $\vartheta \in \Theta_T(M)$ for fixed $w$; or over $(w, \vartheta) \in \mathbb{R} \times \Theta_T(M)$. The space $\Theta_T(M)$ of permissible integrands imposes a square-integrability condition on the stochastic integral process $\int \vartheta_t dS_t$, and the argument $M$ in brackets indicates the existence of trading constraints in the sense that $\vartheta_t(\omega)$ must lie in a convex closed subset $M(\omega, t)$ of $\mathbb{R}^n$. Czichowsky and Schweizer contribute the fact that this setup is the most general formulation for MV hedging under constraints (for evidence of a congruent NHF trading strategy, see Kajiji and Forman (2013)).

### 3.2 The Lexicographic Goal Programming Algorithm

Goal programming is useful for decision analysis where unwanted deviations from a goal are ordered by priority levels in any multi-commodity network (Novikova and Pospelova, 2002). Extant literature has long established that this lexicographic (or pre-emptive) approach to goal programming can be solved by a series of linear programs (Lee, 1972; Ignizio, 1976; Jones and Tamiz, 2010). In the hierarchical goal program we define $C_p$ as the cost vector for goals at the $p$-th priority level for a total of $P$ priorities. The MINLGP solves for the lexicographically smallest vector $(c_1x_1, c_2x_2, \ldots, c_nx_n)$ given $\Gamma_p = \{ x \mid Ax = b, x \geq 0 \}$. The goal program proceeds by solving the first linear program $LP_i = \text{Min} \{ C_i x \mid x \in \Gamma_i \}$ with optimal solution at $x^*$ followed by each $P-1$ remaining $x^*$, that is, the goal program is designed to iterate through all hierarchical priority levels. We close this section by providing a succinct, but complete, mathematical statement of the hierarchical MINLGP:

$$\text{MINLGP} = \text{Min} \left[ C x \mid x \in \Gamma \right],$$

which implies that $\forall x \in \Gamma_p = \{ x \in \Gamma_p \mid C_p x = C_{p, x} x^* \}, \quad p = 1, 2, \ldots, P$. 

3.3 MINLGP and Convex MV Optimization

Beginning with the simplified statement of the bi-criteria MV portfolio optimization model offered by Sharpe, researchers from operations research to computational finance have sought to exploit the convex structure of this simplified model. The quest was driven by an increasing desire among NHF and HF analyst to reduce the computational complexity when solving for optimal solutions under the full covariance specification. The search for simplicity is fairly extensive but also complicated by attempts to join linear specification with multiple objectives. For example, Mansini et al. (2003) provide a detailed summary of different linear programming (LP) solvable models in portfolio optimization. In a similar, but disparate approach, Anagnostopoulos and Mamanis (2010) provide comparative results from solving portfolio models by alternative nonparametric evolutionary search algorithms. However, it is Bartosz (2013) who significantly extends these reviews by singularly targeting multi-criteria portfolio optimization in both a linear and mixed-integer specification. We note, however, that the review of multi-objective algorithms provided by Bartosz did not present any algorithm that combined nonlinear goal- and mixed-integer programming methods.

The newly designed MINLGP processes a bi-criteria Sharpe portfolio model for a finite set of securities where each security is described by a finite and discrete distribution of historical data. Specifically, paraphrasing and extending the notation of Ogryczak (2000), let $X = \{x_1, x_2, \ldots, x_n\}$ denote the set of securities assigned to the model. For each security $x_j \in X$ there is a vector of returns data $(r_{i,j})$ for $i = 1, 2, \ldots, t$ where $r_{i,j}$ is the observed and forecasted rate of return at time $i$ for security $j$. For $t$ security outcomes the data forms a $t \times n$ matrix $R = \{(r_{i,j}); i = 1, \ldots, t; j = 1, \ldots, n\}$ where columns correspond to securities and rows corresponds to outcomes. In this model we solve for the vector of decision variables (weights), $x^*$, that define the optimal portfolio diversification pattern. This model easily incorporates the addition of a contingent claim contract. To do so, the investor chooses a futures contract, $(e.g., f_j)$, from the set of available contracts, $F = \{f_1, f_2, \ldots, f_F\}$, to hedge the systematic risk of $x^*$. Upon selection of a specific contract, the variance minimizing hedge is executed resulting in the determination of a non-fractional number $(N_f)$ of contracts, $f$, to sell against $x^*$. Given the applied motivation driving this research, it is useful to formalize the mathematical statement of the MINLGP algorithm in a manner that allows a clear statement of its potential domain of applicability.

As proposed here, the MINLGP algorithm follows a three-step sequence. The first step involves defining a set of decision variables to characterize the choices to be made. The second step is to add a descriptive goal-directed constraint set. The third step is where the hierarchical goal objective function is stated. More succinctly, we define the MINLGP algorithm as:

$$MINLGP = \text{Min } Z = \text{Min } \sum_{p=1}^{P} Z_p \left[ \delta_p^h h^- + \delta_p^h h^+ \right],$$

S.T. $Ax + Bf + lh^- - lh^+ = b$

$x, f, h^-, h^+ \geq 0$, $f \in \mathbb{Z}_F$, and, $x \in \mathbb{R}^{n-F}$
where \( m \) is the number of constraints such that \( A \in \mathbb{R}^{m \times n}; B \in \mathbb{Z}^{m \times p}, b \in \mathbb{R}^{m} \), and \( Z \) quantifies the \( p \) hierarchical objectives (goals) so that \( Z_1 > Z_2 > \ldots > Z_p \). The constant terms \( \delta_1^p \) and \( \delta_2^p \) are used to indicate relative preference within each \( p \)-th goal. We observe that if \( y = 0 \), then there are no integrality constraints; hence, we obtain the solution to the convex goal program. In goal programming \( b \) is an \( m \)-component vector of goal targets; \( h^- \) and \( h^+ \) are \( m \)-component column vectors that capture goal under- and over-achievement, respectively. Solutions that achieve a minimization of unmet goal hierarchy as much as possible are referred to as feasible solutions. An optimal solution to the convex MINLGP, \( x^* \), is one that satisfies all constraints and meets all goals. Next, we examine the effect on convexity from adding the if-then constraint to the portfolio optimization problem.

3.3.1. If-then Constraints

When there are two or more continuous variables, it is possible to enforce an if-then constraint with the aid of a binary variable. Although the introduction of a binary variable makes the optimization problem non-convex, in section 3.3.2 we introduce a restricted basis entry modified LP that will reconcile the ability of the mixed-integer formulation to generate an optimal solution. Here, the focus is directed to uncover the role of the if-then constraint.

As discussed above, to hedge the portfolio defined by \( x^* \) with a contingent claim hedge the investor must sell quantity \( N_j \) of contract \( f \). The decision to choose a hedging derivative (futures contract) is made based on the investor’s forecast of next period price for that contract, \( f_{t+1} \). In the case where the forecasted price of the liquid futures contract is described by \( f_{t+1} < f_t \) then the number of futures contracts to sell short \((- N_j)\) is enforced by adding an integrality constraint. We provide Theorem 1 to define the constraint’s relationship to \( x^* \). We note that the financial underpinnings of the variance minimizing hedge ratio \((N_j)\) are defined within the context of the proof.

**Theorem 1:** (If-Then): The following three constraints and one binary variable are sufficient to program the If-then statement: IF \( (f_{t+1} - f_t < 0) \) THEN \( \left( N_f = -\beta_p \cdot \frac{V}{f_t} \geq 0 \right) \) where \( V_p \) is the current value of the investment portfolio, \( f_t \) is the value of the futures contract, \( \beta_p \) captures the systematic risk of the portfolio, and the negation indicates a sell decision. For simplicity, let \( A = (f_{t+1} - f_t) \) and \( B = N_f \).

Let binary variable \( v \in \{0,1\} \), and \( |A| \leq N_A \) and \( |B| \leq N_B \).

**Constraints:**
1. \( vN_A \geq A \)
2. \( A + (1 - v)N_A \geq 0 \)
3. \( B \geq -vN_B \)
Proof.

Case 1: If $A > 0$: The first constraint makes $v = 1$. The second constraint has a trivial solution. And, the third constraint becomes $B \geq -N_B$, which, under the assumption that $|B| \leq N_B$, is likewise trivially satisfied.

Case 2: If $A < 0$: The second constraint is satisfied only when $v = 0$. The third constraint now becomes $B \geq 0$.

Case 3: If $A = 0$: the first and second constraints are always satisfied. Taking $v = 1$ we can see that the third constraint is also satisfied.

3.3.2. Convexity and Restricted Basis Entry Separable Programming

The Sharpe diagonal model is defined by $n$ expected returns and $n+1$ variances. The absence of covariance terms makes it straightforward to linearize the model for solution by convex separable programming methods. As a separated linear program, the model is approximated with arbitrary specification in linearization which requires a modified LP that limits basis entry. Additionally, the approximation is known to increase the space complexity of the linearized specification. To iterate to a feasible solution, the separated lexicographic MINLGP proceeds by solving a sequence of modified linear programs such that all priority levels observe branch-and-bound constraints as well as lower- and upper-bound $[0,u]$ separable programming constraints (for the robustness of separable programming, see Feijoo and Meyer (1988), Stefanov (2001), and Jensen and Bard (2002)). Stated differently, the algorithm forms a piecewise linear approximation of each non-convex function by using $d$ line segments. Under this approach to optimization it is necessary to select $d + 1$ values of the scalar $x$ over its range of $0 \leq x \leq u$. By the use of a grid of $d+1$ points for each variable $x_j$ over its range, the separable programming problem in $x$ becomes the following modified linear program in $\alpha$ where an adjacency criterion is imposed on the new decision variable $\alpha_{kj}$ by use of a restricted basis entry rule:

$$
\min f(\alpha) = \sum_{j=1}^{n} \sum_{k=0}^{d} \alpha_{kj} f_{kj}(\bar{x}_{kj})
$$

S.T.:

$$
\sum_{j=1}^{n} \sum_{k=0}^{d} \alpha_{kj} g_{kj}(\bar{x}_{kj}) \leq b_i
$$

$$
\sum_{k=0}^{d} \alpha_{kj} = 1
$$

where: $\alpha_{kj} \geq 0$, $k = 0,...,d$, $j = 1,...,n$, $i = 1,...,m$

In the absence of managerially defined constraints and goals, it is only natural to ponder if the separated linearized Sharpe MINLGP solution is equivalent to the classic Markowitz MV solution obtained by solving a quadratic program. In section 4.4 of this research we perform this comparison. The evidence from the normative application clearly favors the conclusion of near equivalent solutions.
3.4 Trading Strategies and Wealth Creation

In this section we seek to identify how best to implement the Sharpe MINLGP model in an actual trading strategy. Haugh and Jain (2011) provide a concise summary of various contemporary trading strategies. Their classification scheme divides trading strategies into three major styles: static, myopic and a generalized buy-and-hold (GBH). Without summarizing each in detail, we argue that the Sharpe MINLGP portfolio optimization model inherits characteristics from both the static and GBH strategies. The static strategy is defined as a constant proportion weighting scheme where the portfolio weights of risky securities do not vary with time or changes to state variables. Stated differently, this is a trading strategy that does not depend on the predictability of stock prices and, to amplify further, it is a strategy that is defined using unconditional mean returns at time $T^R$ instead of time-varying conditional expected returns on assets. Of course, as time passes the static strategy becomes a dynamic one as securities experience day-to-day variation. This fact causes the static strategy to define some time period, say time, $T^R+I$, as the point when the portfolio is rebalanced with the goal of restoring constant vector proportions. There are many reasons to explain why an investor would adopt a static trading strategy when security prices are known to change dynamically. These reasons include, but are not limited to, the perceived impact of market liquidity conditions, transaction fee assignments, and more.

By contrast, the GBH strategy is defined as an approach where terminal portfolio wealth is solely a function of terminal security prices. Using this approach, the investor assumes responsibility for determining terminal security prices and the asset weighting scheme for the portfolio. For completeness, we note the method by which the investor derives terminal security prices (i.e., fundamental analysis, discounted cash flow, etc.) is not debated in this paper.

The vision for the temporal trading strategy invoked in this research starts by forming one or more portfolios on a decision date ($T^R$). Although security prices vary continuously during the trading day and over the defined trading interval, the impact on the portfolio weighting scheme is left unattended until the adjacent rebalancing date is reached at time $T^R+I$. Under this trading strategy, the time period between $T^R$ and $T^R+I$ is referred to as the interim investment period. Although the equity portfolio is subject to a GBH policy during the interim investment period, the contingent claim contract (e.g., a futures contract) is actively traded within this period. In the next section we explicitly add a constraint which will establish the MVHR as the functional form by which to derive the optimal number of futures contracts ($N_j$) to sell short. In the absence of basis risk the active trading of a hedging instrument will offset any decline to the value of the underlying portfolio. The normative results of this approach are explored in section 4.

4. Application

In this section of the paper we examine the computational tractability of the MINLGP algorithm by executing a temporal trading strategy using a single-period Sharpe efficient
portfolio with $N_j$ contingent claims attached. To fully explore the comparative nature of hedged and unhedged portfolio performance, it is useful to define the parameterized Sharpe model.

The Sharpe single-index model is a simple one-factor model that is designed to explain the expected returns for individual securities. The most recognizable ex-post single index model in the literature is the capital asset pricing model (CAPM): $r_{jt} - r_{ft} = \beta_{jm} (r_{Mt} - r_{ft}) + \varepsilon_{jt}$, where $r_{jt}$ is the interval return at time $t$ for the $j$-th security; $r_{ft}$ is the return on the proxy for the risk-free rate; $r_{Mt}$ is the return on the common market factor, $\beta_{jm}$ is the unit measure of $j$-th securities co-movement with the market factor; and $\varepsilon_{jt}$ captures idiosyncratic risk for the individual security.

### 4.1 Data

The tickers used in the study are presented in table 1 along with the identifier for the market proxy (the S&P 500 Index). Along with the market proxy we obtain historical daily price data for $n, n \in \{1, \ldots, 10\}$, instruments over the period from Nov 1, 2011 through October 31, 2012, inclusive. Using the observed price data, period returns are calculated using log differences for each ticker and the market proxy: $r_{jt} = \ln(P_{t+1}) - \ln(P_t)$.

With parameter estimates in hand, a formal statement of the Sharpe MINLGP model is presented in the next section. Unlike the case where the portfolio optimization is stated as a univectorial objective quadratic program, the hierarchical goal model of the classic Sharpe model requires at least two hierarchical objectives, but, the format is capable of expressing any number of objective priorities that tie together any number of goal constraints.

**Table 1: Estimated Parameters: Sharpe Model**

<table>
<thead>
<tr>
<th>Tickers</th>
<th>$E(r)$</th>
<th>$\sigma^2$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>0.001554</td>
<td>0.000284</td>
<td>0.9218</td>
</tr>
<tr>
<td>ABT</td>
<td>0.000907</td>
<td>0.000086</td>
<td>0.5948</td>
</tr>
<tr>
<td>GOOG</td>
<td>0.000550</td>
<td>0.000226</td>
<td>0.7954</td>
</tr>
<tr>
<td>HD</td>
<td>0.002247</td>
<td>0.000152</td>
<td>0.7776</td>
</tr>
<tr>
<td>IBM</td>
<td>0.000273</td>
<td>0.000128</td>
<td>0.8142</td>
</tr>
<tr>
<td>K</td>
<td>-0.000005</td>
<td>0.000102</td>
<td>0.3061</td>
</tr>
<tr>
<td>PEP</td>
<td>0.000505</td>
<td>0.000058</td>
<td>0.4525</td>
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<td>PFE</td>
<td>0.001176</td>
<td>0.000093</td>
<td>0.6236</td>
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<td>WHR</td>
<td>0.002728</td>
<td>0.000603</td>
<td>1.3568</td>
</tr>
<tr>
<td>YHOO</td>
<td>0.000295</td>
<td>0.000211</td>
<td>0.7082</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td></td>
<td>0.000094</td>
<td></td>
</tr>
</tbody>
</table>
4.2 The Classic Sharpe MINLGP Model

Equation (1) and (2) state the unsystematic and systematic risk goals, respectively. Equation (1) expresses the unexplained variation for \( n \) investment securities, \( \sigma_j^2 \), plus the market proxy, \( \sigma_M^2 \) to account for \( n+1 \) model securities. Structural systematic risk is expressed by equation (2). As required by the Sharpe formulation, this constraint requires the portfolio beta to equal the weighted sum of the individual security beta coefficients. Equation (3) forces the portfolio to be fully invested (no short-sales). Equation (4) is the goal constraint used to set the required return for the efficient portfolio, \( R_p \). Equation (5) is an accounting restatement of portfolio return.

\[
\text{Min: } P_1 \left[ h^- \right] + P_2 \left[ h^+ \right] \\
\sum_{j=1}^{n+1} \sigma_j^2 x_j - h^+_i = 0 \\
\sum_{j=1}^{n} \beta_j x_j = \beta_M \\
\sum_{j=1}^{n} x_j = 1 \\
\sum_{j=1}^{n} E\left(r_j\right) x_j + h^-_i - h^+_i = R_p \\
C_p = \sum_{j=1}^{n} x_j r_j
\]

Equation (0) states the hierarchical objective function. The first priority, \( P_1 \), which must be achieved before consideration of any lower level priorities, minimizes under-achievement of two lower order deviational terms. Both first-order deviational terms have equal weight within this priority level. The first sub-objective sets the required rate of return for the efficient portfolio. The second sub-objective controls the execution of the combinatorial relationship defined by the hedge constraint. The second priority (\( P_2 \)) seeks to minimize over-achievement in the portfolio risk dimension (minimize portfolio risk at the target return level). When taken together, these two objectives operate to fix the specified rate of return and then find the optimal allocation of investment-assets while simultaneously determining \( N_j \), given the forecasted economic conditions. Lastly, we note that for separable programming purposes we replace each nonlinear and non-convex function with a piecewise linear approximation by arbitrarily choosing ten points over the domain of interest [0,1] to account for the proportions invested in any security (note: the solution is available with as few as two points). For the hedging variable the domain scales the set [0,\( M \)] where \( M \) is an arbitrarily large number obtained by substitution of the MVHR as defined below in section 4.3.
4.3 Hedged Sharpe MINLGP Portfolios

The objective of hedging is to offset a loss if wealth due to price volatility in the broad-based market. In this section we expand the Sharpe MINLGP model to include goal constraints to implement a MVHR with respect to the underlying portfolios market value and market beta. The economic condition that drives the decision to hedge is determined exogenously and based solely on the predicted futures contract price for the next period. When the measured relationship is \( \hat{f}_{t+1} < f_t \), the MVHR is calculated and a hedge position is opened. Conversely, when a hedge position is opened and the price relationships are reversed, the hedged position is closed by offset. To begin, here we let \( A = f_t - \hat{f}_{t+1} \); \( B \) is as previously defined; \( L_T \) is the acceptable loss threshold; \( v \) is the binary decision variable, \( v \in \{0,1\} \); and, let \( M \) is an arbitrarily high number. Equation 6 captures the expected dollar loss from volatility of the futures contract. Equation 7 states the managerially determined loss threshold. Whenever the expected loss in value of the futures contract \( (x_f) \) is greater than \( L_T \), \( v = 1 \). In turn, equation 8 executes a sale of \( N_f \) futures contracts.

\[
\begin{align*}
x_f &= A \quad \text{(6)} \\
x_f - Mv + h^-_f &= L_T \quad \text{(7)} \\
N_f - h^+_f &= Bv \quad \text{(8)}
\end{align*}
\]

The importance of thwarting small to unwarranted hedges causes the deviation variable associated with the loss threshold, \( h^- \), to be added to priority 2. The binary hedge decision is treated as a third-priority goal. In a similar manner, the objective function is modified to include minimization of \( h^+_f \) at \( P_3 \). The new objective function, which replaces equation (0), is stated as equation 9.

\[
\begin{align*}
\text{Min} : P_1 \left[ h^-_f \right] + P_2 \left[ h^+_f + h^-_f \right] + P_3 \left[ h^+_f \right]
\end{align*}
\]

4.4 The Equally Weighted Portfolio

The equally weighted portfolio, both hedged and unhedged, requires \( n \) additional constraints. That is, each additional \( i-th \) constraint, \( \{i = 1, \ldots, n\} \), is specified as shown in equation 8 with restriction.

\[
\sum_{j=1}^{n} x_j = (1/n) \quad \text{where} \quad x_j \begin{cases} 1 & \text{if } (i=j) \\ 0 & \text{if } (i \neq j) \end{cases}
\]

The equality relationship for these constraints does not require any adjustment to the objective function. With a completed statement of the Sharpe MINLGP model, we turn our attention to observing a trading strategy and its impact on the wealth creation process under interim period hedging.
4.5 Temporal Investment Portfolio Rebalancing with Interim Period Trading

Studying the benefits of diversification of small and large portfolios using alternative weighting schemes, Daryl and Shawn (2012) found the benefits of diversification favored small equally weighted portfolios compared to market capitalized weighted portfolios during both ‘crisis’ and ‘non-crisis’ economic scenarios. The authors also report a crossover effect as portfolio size increased. These results are relevant to the study of the short investment period as the hypothetical test portfolios are small portfolios that are, one the one hand, equally-weighted and, on the other, optimally diversified.

Over any investment period, the Sharpe MINLGP hedged portfolio combines characteristics of a static trading strategy with those of the GBH. For the purposes of this study, we define portfolio rebalancing time intervals as $T^R, T^{R+1}, \ldots, \infty$. Between any two successive rebalancing dates, the diversified portfolio inherits the characteristics of the GBH portfolio. By definition, the efficient portfolio is formed based on the weighting of security expected returns. However, for the time frame between rebalancing periods, (e.g., between $T^R$ and $T^{R+1}$), we assume the investor actively trades the hedging instrument. The investor observes equally spaced interim periods such that $T^R < T^R + \partial < T^R + 2\partial < \ldots, < T^R + 1$. Based on economic conditions at time $\partial$ the solution to the hedging constraint creates an active trading strategy by executing a transaction of size $N_j$.

The results of solving the efficient sets and various hedged portfolio is presented below. Because the interim hedging periods between static holding period $T^R$ and $T^{R+1}$ is relatively short, we continue the analysis with the static MVHR. The hedged portfolios experience coincident reinvestment of hedge gains (losses) as each hedge is closed.

4.5.1 Comparative Efficient Sets

Since the mid-1970s researchers have sought to compare the efficient sets generated by alternate statements of the portfolio optimization problem. In one early study Frankfurter, et al. (1976) found that the Sharpe diagonal model generated an efficient frontier that differs from the Markowitz efficient set in the region of lower expected returns. Before examining portfolio hedge results, we re-examine the Frankfurter findings by computing two alternate efficient sets. One set is the Markowitz efficient set obtain by application of Lemke’s complementary slackness algorithm (Cottle et al., 1992). The other set is the efficient sets produced by solving the Sharpe MINLGP model as defined above. Additionally, we solve for a specific efficient portfolio. It is the portfolio created by adding the equally weight constraint set as defined in section 4.4.

By observation, the efficient frontiers displayed in figure 1 show that only among the low expected rate of return portfolios is there any observable separation between the two sets. The point labeled “Equally Weighted Portfolio” is the constrained portfolio created by adding the equal weight constraint set. This portfolio has expected return and risk measure of 10.2%; a risk measure of 0.809 (standard deviation units); and, a portfolio beta of 0.740, respectively. The
nearest corner portfolio has measures of 11.4%; 0.684; and 0.591, respectively (see Sanghvi and Dash, (1978) for the definition of a corner portfolio). Relying upon the spatial distances between the two alternatively derived efficient sets, these findings provide evidence that the Sharpe MINLGP solutions are consistent with comparisons to Markowitz derived results in the extant literature.

![Figure 1: Markowitz and Sharpe Efficient Frontiers](image)

4.5.2 **Hedging and Temporal Rebalancing of Investment Portfolios**

The investment period encompasses five trading days: 01-Nov-2012 to 07-Nov-2012 (non-trading weekend days excluded). For performance reporting the test portfolios are tracked from 31-Oct-2012; a date that is referred to as the *base period*. Four portfolios are tracked through the indicated investment period. Two equally weighted portfolios (one hedged and one unhedged) and two diversified optimal portfolios (hedged and unhedged). This approach facilitates a cross-comparative analysis of individual portfolio performance metrics. Before presenting the comparative performance results over the investment period it is useful to examine the portfolio weighting allocations at the re-balancing dates. For convenience, as shown in table 2, we use the notation (M) to identify the equally weighted (or managed) portfolio. The designation of ‘O’ is associated with the portfolio weighting obtained by solving the Sharpe MINLGP model of section 4.2.

On the rebalance date of 08-Nov, the investor would be required to perform a sequence of equity trades in order to maintain the targeted weighting goals for both portfolio (M) and (O). To exemplify this point we focus on ticker HD. To maintain the equally weighted goal set for portfolio (M), a sell order would be executed in the amount of shares required to lower the portfolio weight of ticker HD to 10.00%. For portfolio (O), the investor would purchase shares in an amount that would increase the weight of this security to the new target of 32.92% from its base allocation of 25.24%.
Table 2: Portfolio Composition at Base Period and at Rebalancing Period

<table>
<thead>
<tr>
<th>Tickers</th>
<th>Portfolio M (Oct 31)</th>
<th>Portfolio M (Nov 8)</th>
<th>Portfolio O (Oct 31)</th>
<th>Portfolio O (Nov 8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>9.99%</td>
<td>9.92%</td>
<td>1.48%</td>
<td>1.51%</td>
</tr>
<tr>
<td>ABT</td>
<td>10.00%</td>
<td>9.86%</td>
<td>17.46%</td>
<td>16.12%</td>
</tr>
<tr>
<td>GOOG</td>
<td>10.02%</td>
<td>9.77%</td>
<td>1.48%</td>
<td>1.51%</td>
</tr>
<tr>
<td>HD</td>
<td>10.00%</td>
<td>10.09%</td>
<td>25.24%</td>
<td>32.92%</td>
</tr>
<tr>
<td>IBM</td>
<td>10.01%</td>
<td>10.00%</td>
<td>25.24%</td>
<td>32.92%</td>
</tr>
<tr>
<td>K</td>
<td>10.00%</td>
<td>10.44%</td>
<td>8.93%</td>
<td>6.88%</td>
</tr>
<tr>
<td>PEP</td>
<td>10.00%</td>
<td>10.11%</td>
<td>24.56%</td>
<td>20.82%</td>
</tr>
<tr>
<td>PFE</td>
<td>10.00%</td>
<td>9.97%</td>
<td>22.33%</td>
<td>21.76%</td>
</tr>
<tr>
<td>WHR</td>
<td>10.00%</td>
<td>10.14%</td>
<td>25.24%</td>
<td>32.92%</td>
</tr>
<tr>
<td>YHOO</td>
<td>10.00%</td>
<td>10.42%</td>
<td>25.24%</td>
<td>32.92%</td>
</tr>
</tbody>
</table>

Note: Portfolio M is the equally weighted portfolio; Portfolio O is the optimal portfolio

4.5.3 Hedged Portfolios

Tables 3 and 4 display the dollar denominated results of the interim period hedge for each portfolio. Before describing the outcomes, we note that the dollar difference in total portfolio value is directly attributed to the disparate weighting apportionment.

Table 3: Hedged Portfolio (M)

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov 1</td>
<td>$865,401</td>
<td>$1,417</td>
<td>$70,850</td>
<td>9</td>
<td>$637,650</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nov 2</td>
<td>$857,368</td>
<td>$1,399</td>
<td>$69,962</td>
<td>9</td>
<td>$629,662</td>
<td>$7,987</td>
<td></td>
</tr>
<tr>
<td>Nov 5</td>
<td>$860,909</td>
<td>$1,405</td>
<td>$70,287</td>
<td>9</td>
<td>$638,550</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nov 6</td>
<td>$863,270</td>
<td>$1,419</td>
<td>$70,950</td>
<td>9</td>
<td>$638,550</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nov 7</td>
<td>$851,363</td>
<td>$1,382</td>
<td>$69,137</td>
<td>9</td>
<td>$622,237</td>
<td>$16,312</td>
<td></td>
</tr>
<tr>
<td>Position</td>
<td>$(14,038)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$34,299</td>
</tr>
</tbody>
</table>

Table 4: Hedged Portfolio (O)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov 1</td>
<td>$2,164,294</td>
<td>$1,417</td>
<td>$70,850</td>
<td>16</td>
<td>$1,133,600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nov 2</td>
<td>$2,151,263</td>
<td>$1,399</td>
<td>$69,962</td>
<td>16</td>
<td>$1,119,400</td>
<td>$14,200</td>
<td></td>
</tr>
<tr>
<td>Nov 5</td>
<td>$2,153,150</td>
<td>$1,405</td>
<td>$70,287</td>
<td>16</td>
<td>$1,135,200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nov 6</td>
<td>$2,160,581</td>
<td>$1,419</td>
<td>$70,950</td>
<td>16</td>
<td>$1,135,200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nov 7</td>
<td>$2,144,585</td>
<td>$1,382</td>
<td>$69,137</td>
<td>16</td>
<td>$1,106,200</td>
<td>$29,000</td>
<td></td>
</tr>
<tr>
<td>Position</td>
<td>$(19,709)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$43,200</td>
</tr>
</tbody>
</table>
Upon solving the Sharpe MINLGP at time $T^R$, the *if-then* hedging constraint, equation 6, produced a solution to sell 9 E-mini S&P 500 futures contracts against portfolio (M). For portfolio (O) the optimal solution is to sell 16 contracts. The active interim period futures hedging strategy up to the next rebalance date at time $T^R+1$ is to dynamically open/close a futures position in the amount of $N_f$ contracts based on next period ($\hat{\sigma}_{t+1}$) forecasted values of $f$. Because the futures hedge is designed to value only the futures contract both portfolios experience active hedge activity on the same days. Over the simulated interim period two dynamic hedges were opened and closed by offset. Based on the number of contracts involved, the actual dollar profit is shown in tables 3 and 4, respectively. For clarity of exposition, tables 3 and 4 do not show coincident reinvestment of gains (losses) from futures trading.

Over the investment period both portfolios (M and O) experienced a loss in dollar value of $14,308 (1.62\%)$ and $19,709 (0.9\%)$, respectively. As expected, between rebalancing dates, the interim period hedging gains of $34,299$ and $43,200$ more than offset the decline in respective portfolio values to generate a weekly return of $2.34\%$ and $1.09\%$.

### 4.5.4 Comparative Risk Performance

Figures 2a to 2d present grouped bar chart of comparative risk measures. We note that the base group is dated 31-Oct-2012 (not included in tables 3 and 4). The remaining groups are the trading days 01-Nov-12 through 07-Nov-12, excluding weekend days. The legend shows results for two hedged portfolios: one is the equally weighted (MH) and the other is the optimal portfolio (OH). For reference, the risk profiles of two unhedged portfolios are also displayed: one for the equally weighted (MU) and one for the optimal portfolio (OU). For this study we focused on four risk metrics. The Sharpe ratio (figure 2a) reports the ability of a portfolio to generate risk-adjusted returns per unit of total risk. As a modified Sharpe ratio, the Sortino performance metric (figure 2b) penalizes only those returns falling below a managerial determined target return (Sortino, 2009). The Sharpe-Omega metric (figure 2c) takes into account the asymmetry of the return probability distribution (Hentati-Kaffel and Prigent, 2012). Lastly, in figure 2d we present VaR (value-at-risk) at the 5\% probability level. VaR indicates the potential for losses on the portfolio for a stated time horizon (Jorion, 2006). The comparative results as they related to these four risk measures follows.

The Sharpe ratio groupings maintain a consistent view across all reported time frames. Interestingly, the Sharpe ratio for the equally weighted hedged portfolio is greater than the Sharpe ratios for all other portfolios. This finding is consistent with reported findings that favor small equally weighted portfolios to generate comparatively high Sharpe ratios. As previously defined, the Sortino ratio only penalizes downside portfolio risk. Unlike the Sharpe ratio results, the Sortino risk-adjusted rankings clearly favor the optimal portfolio – hedged and unhedged. The Sharpe-Omega metric follows the Sharpe ratio analysis without deviation. Over the investment period both the hedged and unhedged equally weighted portfolio achieves higher risk-adjusted performance.
Portfolio VaR focuses on the risk of an asset rather than its return. The VaR metric is always expressed in the units and time-frame of the underlying instrument. For comparative balance, figure 2d displays the daily-dollar denominated VaR as a percent of portfolio value (rather than the raw metric). For each interim time period and for all portfolios, hedged and unhedged, the VaR of the optimally diversified portfolio shows little variation in the percentage of portfolio value at risk. We also note that the VaR for the optimally diversified portfolio is lower than all other portfolio VaRs. The VaR for the unhedged equally weighted portfolio is consistently the highest across all periods and hedge styles.
5. Summary and Conclusions

The research presented in this paper was inspired by investors who seek to hedge efficient portfolios as a means by which to improve Sharpe ratios. To that end, the paper successfully combined elements of mixed-integer programming with those of nonlinear multi-objective programming to produce a convex separable-programming based MINLGP. To demonstrate the algorithm’s domain of applicability a portfolio theoretic wealth building example was designed, solved and examined across consecutive daily observations. Policy outcomes that conform to and extend extant findings were reported.

First, except for low rate of return portfolios, the results confirmed the ability of the new MINLGP algorithm to produce an efficient set of portfolios generated from a bi-criteria Sharpe model that nearly replicates one produced by a uni-objective quadratic program using the full covariance matrix. From this exercise it is clear that when using a lexicographic goal programming specification, the near-identical efficient set solution is generated by specification of two hierarchical priority levels – one to minimize deviation from the required portfolio return and another to minimize overachievement of the second-order risk goal.

Second, prior findings in the literature were amplified upon reviewing the temporal risk-adjusted performance of small sample portfolios after solving the MINLGP and then hedging these same portfolios across the interim period between portfolio rebalance dates. The results indicate that risk-adjusted performance for a small equally weighted portfolio did not differ by a large amount when compared to a small efficiently diversified portfolio, hedged or unhedged. Both the risk-adjusted return performance metrics and the VaR percentile method confirmed that, in the absence of MVHR-based hedging, investors can hold small equally diversified portfolios to obtain many of the benefits offered by a well diversified portfolio. Lastly, the study of comparative portfolio performance also established the benefits from hedging using the MVHR on a daily basis with the E-mini S&P 500 futures contracts. Hedged portfolios generated measureable improvements in overall risk-adjusted return and catastrophic risk management using VaR.

These results add new information about the usefulness of interim period hedging when optimal rebalancing is performed on a weekly time scale with daily interim period hedging using a traded futures contract. However, the findings in this study also raise new research questions. For example, the temporal delineation or the portfolio rebalance period deserves a more detailed study. That is, in addition to testing a longer rebalancing time frame (e.g., multiple weeks, months, etc.), it is equally important to provide a clear distinction between small, medium and large portfolios. Unarguably, a ten ticker portfolio is a small portfolio. But, do the benefits of hedging as found in this study hold when portfolio size increases? Lastly, we note that the static variance minimizing hedge ratio performed profitably across the sample investment period. However, we also note that the ratios consistently over-sold futures contracts. Had any one of the hedges faced significant basis risk, the unintended results could have been devastating. These observations provide a justification to examine alternate hedge ratio specifications. We leave these questions for future research.
References


Founded in 1892, the University of Rhode Island is one of eight land, urban, and sea grant universities in the United States. The 1,200-acre rural campus is less than ten miles from Narragansett Bay and highlights its traditions of natural resource, marine and urban related research. There are over 14,000 undergraduate and graduate students enrolled in seven degree-granting colleges representing 48 states and the District of Columbia. More than 500 international students represent 59 different countries. Eighteen percent of the freshman class graduated in the top ten percent of their high school classes. The teaching and research faculty numbers over 600 and the University offers 101 undergraduate programs and 86 advanced degree programs. URI students have received Rhodes, Fulbright, Truman, Goldwater, and Udall scholarships. There are over 80,000 active alumnae.

The University of Rhode Island started to offer undergraduate business administration courses in 1923. In 1962, the MBA program was introduced and the PhD program began in the mid 1980s. The College of Business Administration is accredited by The AACSB International - The Association to Advance Collegiate Schools of Business in 1969. The College of Business enrolls over 1400 undergraduate students and more than 300 graduate students.

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Our responsibility is to provide strong academic programs that instill excellence, confidence and strong leadership skills in our graduates. Our aim is to (1) promote critical and independent thinking, (2) foster personal responsibility and (3) develop students whose performance and commitment mark them as leaders contributing to the business community and society. The College will serve as a center for business scholarship, creative research and outreach activities to the citizens and institutions of the State of Rhode Island as well as the regional, national and international communities.

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