Duration and Convexity

1. Price Volatility
2. Duration
3. Convexity
Price Volatility of Different Bonds

Why do we have this chapter?

-- how bond price reacts to changes in bond characteristics, including the following:

-- Coupon rate: Inversely related to price volatility.

-- Term to maturity: Positively related to price volatility.

-- Yield to maturity: Inversely related to price volatility.
Measures of Price Volatility (1)

(A) Price value of a basis point
also known as dollar value of an 01. It is the change of bond price when required yield is changes by 1 bp.

Example: see page 62

5-year 9% coupon, YTM=9%, initial price=100,
Price at 9.01% is _____; price value of a basis point = Abs(price change) =
Measures of Price Volatility (2)

(B) Yield value of a price change
change in the yield for a specified price change.
the smaller the number, the larger price volatility.
## Example for Yield value of a price change

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Ini yld</th>
<th>Ini Prc</th>
<th>Prc Chg</th>
<th>Yld Chg</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 years</td>
<td>100</td>
<td>9%</td>
<td>1</td>
<td>?</td>
</tr>
<tr>
<td>10 years</td>
<td>100</td>
<td>9%</td>
<td>1</td>
<td>?</td>
</tr>
</tbody>
</table>
(c) Macaulay duration

\[ Macaulay\ _\ Duration = \sum_{t=1}^{n} \frac{tC}{(1 + y)^t} + \frac{nM}{(1 + y)^n} \]

It can be shown that

\[ \frac{dP}{dy} \frac{1}{P} = - \frac{1}{1 + y} * Macaulay\ _\ Duration \]

Where, \[ P = \frac{C}{1 + y} + \frac{C}{(1 + y)^2} + \ldots + \frac{C}{(1 + y)^n} + \frac{M}{(1 + y)^n} \]
Modified Duration

Modified Duration = Macaulay duration / (1 + y)

\[ \frac{dP}{dy} \frac{1}{P} = -\text{modified duration} \]

Modified duration is a proxy for the percentage in price. It can be interpreted as the approximate percentage change in price for a 100-basis-point change in yield.

Look at exhibit 4-12 on page 74: graphic interpretation of duration
Approximating the Dollar Price Change

\[
\frac{dP}{dy} = (-\text{Modified duration})P
\]

dollar duration = (-\text{Modified duration})P
Example

Calculate Macaulay duration and modified duration for a 5-year bond selling to yield 9% following the examples in the text.

What is the dollar change in bond price when interest rate increases by 1%?
Portfolio Duration

Find the weighted average duration of the bonds in the portfolios.

Contribution to portfolio duration: multiplying the weight of a bond in the portfolio by the duration of the bond.

See the example on pages 71-72.
Approximating a Bond’s Duration

Page 84

\[ \text{Approximate Duration} = \frac{P_0 - P_+}{2(P_0)(\Delta y)} \]
Convexity

• Duration is only for small changes in yield or price – assuming a linear relation between yield change and price change
• Convexity is for the nonlinear term
• See the graph on page 75
Measuring Convexity

Taylor Series

\[ dP = \frac{dP}{dy} dy + \frac{1}{2} \frac{d^2 P}{dy^2} (dy)^2 + \text{error} \]

\[ \frac{dP}{P} = \frac{dP}{dy} \frac{1}{P} dy + \frac{1}{2} \frac{d^2 P}{dy^2} \frac{1}{P} (dy)^2 + \frac{\text{error}}{P} \]

Convexity = \[ \frac{d^2 P}{dy^2} \frac{1}{P} \]

where \[ \frac{d^2 P}{dy^2} = \sum_{t=1}^{n} \frac{t(t+1)C}{(1 + y)^{t+2}} + \frac{n(n + 1)M}{(1 + y)^{n+2}} \]
Example

Coupon rate: 6%
Term: 5 years
Initial yield: 9%
Price: 100

(page 76)
Percentage Change in price due to Convexity

\[
\frac{dP}{P} = \frac{1}{2} \text{Convexity}(dy)^2 \quad \text{or} \quad \frac{\Delta P}{P} = \frac{1}{2} \text{Convexity}(\Delta y)^2
\]

Example: see page 76
Implications

High convexity, high price, low yield.

Convexity is valuable.

Convexity is more valuable when market volatility is high. Look at Exhibit 4-12 on page 80.
Approximate Convexity Measure

Page 84

\[
\text{Appr. Convexity} = \frac{P_+ + P_- - 2P_0}{P_0 (\Delta y)^2}
\]
Percentage Price Change using Duration and Convexity Measure

\[
\frac{\Delta P}{P} = -Duration \times \Delta y + \frac{1}{2} Convexity \times (\Delta y)^2
\]

Example see page 78 - 79.
Yield risk for nonparallel changes in interest rates

1). Yield Curve Reshaping Duration
Page 86:  a) SEDUR; b) LEDUR
Formula for both are identical to 4.23 on page 84.

2) Key rate duration (page 86)
Assume in every yield, we have a different duration. Then we have a duration vector.
Key rate durations

- page 87
- Rate duration: the sensitivity of the change in value to a particular change in yield
- Key rate durations (duration vectors) are measured using: 3m, 1 year, 2-year, 3-year, 5-year, 7-year, 10-year, 15-year, 20-year, 25-year and 30-year bonds